Math 301
Quiz 2

Justify all answers with neat and organized work. Clearly indicate your answers. 20 points possible.

1. (5 pts.) Consider the following statement.

$(*)$ $\forall n \in \mathbb{Z}$, if n is prime, then n is odd or $n = 2$.

(a) Write the contrapositive of statement $(*)$.

(b) Write the converse of statement $(*)$.

(c) Write the inverse of statement $(*)$.

(d) Write the negation of statement $(*)$.

2. (5 pts.) State whether the given argument has a valid or invalid form. (You do not have to justify your answers.)

(a) All freshmen must take writing.
Caroline is a freshman.
$\therefore$ Caroline must take writing.

(b) All healthy people eat an apple a day.
Herbert is not a healthy person.
$\therefore$ Herbert does not eat an apple a day.

(c) If a product of two numbers is 0, then at least one of the numbers is 0.
For a particular number x, the product $(x-1)(x+1)$ is not 0.
$\therefore$ Neither $(x-1)$ nor $(x+1)$ equals 0.

(d) All cheaters sit in the back row.
George sits in the back row.
$\therefore$ George is a cheater.

(e) All honest people pay their taxes.
Darth is not honest.
$\therefore$ Darth does not pay his taxes.

3. (5 pts.) Prove this theorem.

Theorem. *The product of any even integer and any odd integer is even.*

4. (5 pts.) Prove this theorem.

Theorem. *For all integers a, b, and c, if $a \mid b$ and $a \mid c$, then $a \mid (b-c)$.*