MTH 162
Exam 1
Spring 2017

Show all work in a neat and organized fashion. Clearly indicate your answers.
100 points possible.

A graphing calculator is allowed (e.g., TI-84). No calculator with a Computer Algebra System (CAS) is allowed (e.g., TI-89, TI-Nspire).

The following formulas may or may not be useful:

$y - y_1 = m(x - x_1)$

$x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}, \qquad x = \dfrac{-b}{2a}$

$\log_a(MN) = \log_a M + \log_a N, \qquad \log_a(M/N) = \log_a M - \log_a N, \qquad \log_a(M^N) = N \log_a M$

$\log_b x = \dfrac{\log_a x}{\log_a b} = \dfrac{\ln x}{\ln b} = \dfrac{\log x}{\log b}$

0. (4 pts.) Four free points.

1. (12 pts.) Perform the indicated operation and simplify.

$$\frac{x^2 + x - 12}{x^2 - 4x + 4} \div \frac{x^2 + 3x + 2}{x^2 - 4}$$

2. (12 pts.) If $f(x) = 5x - x^2$, find $\dfrac{f(x+h) - f(x)}{h}$ and simplify.

3. (12 pts.) For this problem, $y = 3x^2 + 30x$.

(a) Find the vertx of the graph of the equation.

(b) Determine if the vertex is a maximum or minimum point.

(c) Determine what value of x gives the optimal value of the function.

(d) Determine the optimal (maximum or minimum) value of the function.

4. (12 pts.) If a company has total costs $C(x) = 12{,}000 + 25x + 0.1x^2$ and total revenues given by $R(x) = 285x - 0.9x^2$, find the break-even points.

5. (12 pts.) (a) Graph the solution of this system of inequalities.

(b) Find and clearly label all corner points of the solution region.

$$\begin{aligned} 3x + y &\leq 10 \\ x + 2y &\leq 5 \\ x &\geq 0 \\ y &\geq 0 \end{aligned}$$

6. (12 pts.) Bluejay Insulating Company manufactures two types of storm windows: Model H (the heavy duty) and Model R (the regular). To make one Model H window, it requires 4 hours of cutting and 3 hours of finishing. On the other hand, to make one Model R window, it takes 5 hours of cutting and 2 hours of finishing. Production scheduling indicated that during the coming week, there will be at most 30 hours of cutting available, and at most 19 hours of finishing.

Write the system of inequalities that describes the constraints on the number of each type of cake to be made. Begin by identifying what x and y represent.

Don't graph the inequalities or find the corners. Just say what x and y represent, and write the inequalities. That's all.

(Here is additional *optional* information that you do not need to use; I'm only stating this because it might help you understand the setting: Model H sells for \$45 per window, while Model R sells for \$35 per window. The company would want to know how many of each window will maximize revenue; however, don't go on to figure that out.)

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7. (12 pts.) (a) Write the expression as the sum or difference of two logarithms containing no radicals or exponents.

$$\log_4 \frac{x}{x+5}$$

(b) Use the properties of logarithms to write the expression as a single logarithm.

$$\frac{1}{2}\log x + \log(x+8)$$

8. (12 pts.) On a college campus of 20,000 students, a single student returned to campus infected by a disease. The spread of the disease through the student body is given by

$$y = \frac{20{,}000}{1 + 19{,}999e^{-0.75t}}$$

where y is the total number infected at time t (in days).

(a) How many are infected after 5 days?

(b) When will 50% of the students be ill?