Math 162
Exam 1

Show all work in a neat and organized fashion. Clearly indicate your answers.
100 points possible.

Units, Units, Units! Label all numeric answers. With models, give a concise explanation of the variables, including units.

Example: Suppose a problem asks for a model, and suppose this is the answer:

$$Tax = 2538.90 + 540.37t \text{ dollars, where } t \text{ is the number of years since 1989.}$$

To receive full credit, this answer must include **all** of the following:
(1) the correct model (i.e., $Tax = 2538.90 + 540.37t$),
(2) the correct label for the output (i.e., "dollars"), and
(3) the correct explanation of the input variable (i.e., "where t is the number of years since 1989").

Example: Here is another correct way to write the same answer.

$D = 2538.90 + 540.37x$, where D = dollars of tax, and x = number of years since 1989.

Some useful (?) formulas:

$A = P(1 + \frac{r}{n})^{nt}$
$A = Pe^{rt}$
APY (as a decimal) $= (1 + \frac{r}{n})^n - 1$
APY (as a decimal) $= e^r - 1$

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1. (4 pts.) Determine if the table below represents a function. Assume the input is in the left column.

Person's Height (inches)	Person's weight (pounds)
63	139
73	196
64	115
72	203
70	115

2. (6 pts.) In a certain neighborhood, homes are valued at \$55 per square foot, driveways at \$1.50 per square foot, and decks at \$3.25 per square foot.

(a) If you were an appraiser, what mathematical model would you use to determine the value of a home?
(b) Use your model to find the appraised value of a 1750 square foot home with a 325 square foot deck and a driveway that is 12 feet wide by 50 feet long.

3. (4 pts.) $H(w)$ is total cost in dollars for purchasing w shirts. Write the following statement in function notation (like J(1985)=1985.)

To buy 250 shirts, it costs \$750.

4. (4 pts.) Calculate

$$\frac{2.956}{1+4.85e^{-.498}}$$

Round your answer to four decimal places.

5. (4 pts.) Find out for what x value $\dfrac{4000}{1+45e^{-.213x}}$ is equal to 2000. Round your answer to four decimal places.

6. (6 pts.) Consider

$$S(t)=\frac{150}{1+23e^{-.6t}}; \qquad t=2.6$$

(a) Is $t=2.6$ an input or an output value?

(b) Find the output or input that corresponds to $t=2.6$. Round your answer to four decimal places.

7. (4 pts.) Consider this pair of functions.

$R(d)$ = air temperature at an altitude of d feet above sea level.

$D(p)$ = altitude of an airplane in feet above sea level when it has been flying for p minutes.

These two functions can be combined by function composition. Which of the following is the correct new function? $D \circ P$, $R \circ D$, $P \circ D$, $D \circ R$, $R \circ P$, $P \circ R$

8. (4 pts.) (Based on data from Ryan and Gross, "The Diffusion of Hybrid Seed Corn in Two Iowa Communities," *Rural Sociology*, March 1943.) The percentages of Iowa corn farmers in two communities who had heard about, and who had planted, hybrid seed corn t years after 1924 can be modeled as follows.

$$\text{Percentage hearing} = h(t) = \frac{100}{1+128.04e^{-0.72t}} \text{ percent,}$$

$$\text{Percentage planting} = p(t) = \frac{100}{1+913.72e^{-0.61t}} \text{ percent}$$

Write a model for the percentage of Iowa farmers who had heard about but not yet planted hybrid seed corn t years after 1924.

9. (4 pts.) Rewrite this pair of functions as one composite function (using the order naturally suggested by the choice of letters for the variables). You do not have to simplify your answer.

$g(x) = \sqrt{7x^2 - 5x + 8}, \qquad x(w) = 4w^3 - 9$

10. (16 pts.) The revenue for Funky Toy Company was \$150.3 million in 1994 and \$120.7 million in 1995. Assume that revenue is decreasing at a constant rate, so that a linear model applies.

(a) Find the rate of change of revenue. Include correct units for rate of change.

(b) By how much did revenue decline each quarter of 1995?

(c) Complete the following table.

Year	Revenue (millions of dollars)
1994	
1995	
1996	
1997	

(d) Find a linear model for the revenue.

11. (16 pts.) The data in the table below show how postal rates have increased for first-class letters weighing up to one ounce, from 1919 to 1995.

Year of rate increase	Rate (cents)
1919	2
1932	3
1958	4
1963	5
1968	6
1971	8
1974	10
1975	13
1978	15
1981	20
1985	22
1988	25
1991	29
1995	32

(a) Find an exponential model to fit the data. According to this model, what was the postage in 1995?

(b) Discuss how well the model fits the data.

(c) Disregard the first two data points, and find an exponential model to fit the data from 1958 to 1995. According to this model, what was the postage rate in 1995?

(d) Discuss how well this new model fits the data.

(e) Use the two models to predict the postage rate in the year 2009. Would you consider either of these predictions to be a good indicator of what may happen? Explain your reasons.

12. (16 pts.) In 1949, the United States experienced the second worst polio epidemic in its history. (The worst was in 1952.) The table below gives the cumulative number of polio cases diagnosed on a monthly basis.

Month	Total number of polio cases
January	494
February	759
March	1016
April	1215
May	1619
June	2964
July	8489
August	22,377
September	32,618
October	38,153
November	41,462
December	42,375

(a) Observe a scatter plot of the data from January through June only. Describe the concavity indicated by the scatter plot. Does this portion of the data appear to be logistic? [Write down the calculator's "Window" settings, for use in part (d).]

(b) Observe a scatter plot of the entire data set given. Does the entire data set appear to be logistic?

(c) Find a logistic model for the entire data set.

(d) Graph the model from part (c), but use the graphing window from part (a). Discuss how well the model fits this portion of the data.

13. (4 pts.) If a country has a growth rate of 3.4 percent per year, how many years will it take for that country's population to double? Use an exponential model, and please round your answer to 3 decimal places.

14. (4 pts.) You invest \$9000 at 7.5% APR compounded monthly. Find the amount of the investment after 6 years and 3 months.

15. (4 pts.) The formula that is used to calculate the monthly payment of a loan is

$$\frac{r}{12}A = m\left[1 - \left(1 + \frac{r}{12}\right)^{-n}\right]$$

where A is the loan amount, r is the interest rate expressed as a decimal, n is the number of months of the loan, and m is the monthly payment. Suppose you are taking out a \$120,000 home loan at a rate of 6% compounded monthly for 20 years. Determine the monthly payment for this loan.