

Math 162**Exam 2**

Show all work in a neat and organized fashion. Clearly indicate your answers.
100 points possible.

Units! Units! Units! Your answers should include the correct units.

1. (12 pts.) At the end of World War I, in the fall of 1918, an influenza epidemic hit the United States Navy. It spread to the Army, to American civilians, and ultimately to the world. It is estimated that by 1920, twenty million people had died from the epidemic. Five hundred fifty thousand of these were Americans—over ten times the number of WWI battle deaths. The tables below give the total numbers of Navy, Army, and civilian deaths due to the epidemic (A. W. Crosby, Jr., *Epidemic and Peace 1918*, Westport, Connecticut: Greenwood Press, 1976).

Week ending	Total Deaths		Total Civilian Deaths
	Navy	Army	in 45 Major Cities
August 31	2		
September 7	13	40	
September 14	56	76	68
September 21	292	174	517
September 28	1172	1146	1970
October 5	1823	3590	6528
October 12	2338	9760	17,914
October 19	2670	15,319	37,853
October 26	2820	17,943	58,659
November 2	2919	19,126	73,477
November 9	2990	20,034	81,919
November 16	3047	20,553	86,957
November 23	3104	20,867	90,449
November 30	3137	21,184	93,641

(a) How rapidly (on average) did the number of deaths from influenza increase in the Navy between August 31, 1918 and October 12, 1918?

(b) Compare the average rates of increase in the Navy and Army between October 19, 1918, and November 2, 1918.

2. (6 pts.) Describe the difference between average rate of change and instantaneous rate of change.

3. (4 pts.) Let $m(t)$ be the membership of the National Rifle Association (NRA) in millions of people, where t is the number of years since 1990.

Select the sentence that best gives an interpretation of the following:

$$m'(4) = 0.25$$

(based on data from The Associated Press, *Anderson Independent-Mail*, May 20, 1995, page A1).

- A. In 1994, NRA membership was increasing at a rate of 250,000 people per year.
- B. When NRA membership was 4 million people, the number of years since 1990 was increasing at a rate of 0.25 million people per year.
- C. One quarter of a year since the start of 1990, NRA membership was increasing at a rate of 4 million people per year.
- D. During the first 3 months of 1990, NRA membership increased at a rate of 4 million people per year, on average.
- E. In 1994, NRA membership was 0.25 million people.
- F. When NRA membership was 4 million people, it was 3 months into 1990.
- G. From 1990 through 1994, NRA membership increased at a rate of 250,000 people per year, on average.

4. (18 pts.) Predictions for the US resident population from 1997 through 2050, as reported by *Statistical Abstract* for 1994, can be approximated by the model

$$p(t) = 2370.15t + 39,789.96 \text{ thousand people,}$$

where t is the number of years since 1990. A graph of $p(t)$ is shown below.

- (a) Sketch a tangent line at $t = 120$ and find its slope.
- (b) What is true about any line tangent to this model?
- (c) What is the slope of any line tangent to this graph?
- (d) What is the slope at every point on the graph of this model?
- (e) According to the model, what is the instantaneous rate of change of the predicted population in any year from 1997 through 2050?

5. (12 pts.) The graph below shows the number of births (in thousands) in the United States to women under 15 years of age from 1970 through 1991 (based on data from *Statistical Abstract*, 1992).

- (a) Estimate the change and percentage change in the number of such births from 1980 through 1990.
- (b) Estimate the rate of change and percentage rate of change in 1982.

6. (12 pts.) The percentage of households with VCRs can be modeled by the equation

$$P(t) = \frac{72.5}{1 + 75.473e^{-0.6486t}} \text{ percent}$$

where t is the number of years since 1980 (based on data from *Statistical Abstract*, 1994). A graph is shown below.

- Use the graph to estimate $\frac{dP}{dt}$ when $t = 7$.
- Use the equation to investigate $P'(7)$ numerically by choosing at least three increasingly close points.
- Briefly discuss the advantages and disadvantages of using the two methods above.

7. (12 pts.) A graph depicting the monthly profit for Slim's Used Car Sales is shown below.

- Estimate the instantaneous rates of change at 20, 40, 60, 80, and 100 cars. (The answers, in ascending order, are -1185 , 0 , 0 , 160 , and 770 . Just match each point with its correct slope.)
- On the basis of your answers to part (a), sketch a rate-of-change graph. Label the units on the axes.
- For what number of cars sold between 20 and 100 is monthly profit increasing most rapidly? For what number of cars sold is monthly profit decreasing most rapidly? What is the mathematical term for these points?

8. (12 pts.) On the basis of the following information, sketch a possible graph of $t(x)$.

- $t(4) = 12$
- $t(6) = t(10) = 0$
- $\frac{dt}{dx} = 0$ at $x = 8$
- The graph of $t(x)$ has no concavity changes.

9. (12 pts.) Use the four-step method to find the derivative of $y = -6x^2 - 3x + 4$.

(Step 1) Write down $f(x)$.

(Step 2) Find and simplify $f(x + h)$.

(Step 3) Find and simplify $\frac{f(x + h) - f(x)}{h}$. Show all steps of the algebra clearly and neatly.

(Step 4) Find the limiting value of $\frac{f(x + h) - f(x)}{h}$ as h approaches 0.