Math 162
Exam 1

Show all work in a neat and organized fashion. Clearly indicate your answers.
100 points possible.

Units! Units! Units! Your answers should include the correct units. With models, give a concise explanation of the variables.

Example: Suppose a problem asks for a model, and suppose this is the answer:

$$Tax = 2538.90 + 540.37t \text{ dollars, where } t \text{ is the number of years since 1989.}$$

To receive full credit, this answer must include **all** of the following:
(1) the correct model (i.e., $Tax = 2538.90 + 540.37t$),
(2) the correct label for the output (i.e., "dollars"), and
(3) the correct explanation of the input variable (i.e., "where t is the number of years since 1989").

Example: Here is another correct way to write the same answer.

$D = 2538.90 + 540.37x$, where D = dollars of tax, and x = number of years since 1989.

Some useful (?) formulas:

$A = P(1 + \frac{r}{n})^{nt}$
$A = Pe^{rt}$
APY (as a decimal) $= (1 + \frac{r}{n})^n - 1$
APY (as a decimal) $= e^r - 1$

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1. (12 pts.) For each rule, (1) draw an input/output diagram, (2) determine whether the rule is a function, and (3) if the rule is a function, state the set of inputs and the set of outputs for the function.

(a) $B(t)$ = the amount in an investment account (in dollars) after t years, assuming that no deposits or withdrawals are made during the t years

(b) $A(t)$ = the amount in an investment account (in dollars) after t years, assuming that deposits and withdrawals are permissible

2. (6 pts.) Consider

$$P(x) = 0.01244x^3 - 0.24209x^2 + 1.40699x + 3.44455; \qquad P(x) = 4.43$$

(a) Is $P(x) = 4.43$ an input or an output value?

(b) Find the output or input that corresponds to $P(x) = 4.43$. Round your answer to four decimal places.

3. (6 pts.) The selling price (in dollars) of a gallon of milk and the number of gallons of milk sold on the xth day of a certain month are modeled as follows.

$$\text{Selling price} = S(x) = 0.007x + 1.492 \text{ dollars per gallon,}$$
$$\text{Amount sold} = G(x) = 31 - 6.332(0.921)^x \text{ gallons}$$

Write a model for the total sales (in dollars) from milk on the xth day of that month.

(Be sure to give the correct model/formula, the correct label for the units of the output, and the correct explanation of the input variable including units.)

4. (6 pts.) Determine whether the following pair of functions can be combined by function composition. If so, tell whether the function notation for the new function is $C \circ P$ or $P \circ C$, and then draw and label its input/output diagram.

$C(t)$ is the average number of customers in a restaurant on a Saturday night t hours after 4 p.m.

$P(c)$ is the average amount in tips generated by c customers.

5. (6 pts.) Swap the inputs and outputs for the function below. (1) draw an input/output diagram for the swapped rule, (2) write out the swapped rule in words, and (3) determine whether the swapped rule is a function.

$R(w)$ = the first-class postal rate (in cents) of a letter weighing w ounces

6. (10 pts.) The table below gives the number of gallons of oil remaining in a tank used for heating an apartment complex t days after January 1 when the tank was filled.

t	Oil (gallons)
0	30,000
1	29,400
2	28,800
3	28,200
4	27,600

(a) Find a linear equation for the amount of oil in the tank.

(b) What is the rate of change of the amount of oil? (Be sure to include correct units.)

(c) How much oil can be expected to be used during any particular week in January?

7. (10 pts.) Suppose that the population of a certain state between 1986 and 1994 is shown in the table below.

Year	Population (thousands)
1986	669
1987	661
1988	655
1989	646
1990	639
1991	641
1992	642
1993	644
1994	645

(a) Observe a scatter plot of the data. Write down the values of Xmin, Xmax, Ymin, and Ymax that you used (press Window to see these values).

(b) From the scatter plot, what year is the dividing point that should be used to create a piecewise continuous function?

(c) Divide the data in the year you determined in part (b). Include the dividing point in both data sets. Fit linear models to each set of data, and write the function in correct piecewise continuous function notation. Round the numbers to four decimal places.

8. (10 pts.) At the end of World War I, in the fall of 1918, an influenza epidemic hit the United States Navy. It spread to the Army, to American civilians, and ultimately to the world. It is estimated that by 1920, twenty million people had died from the epidemic. Five hundred fifty thousand of these were Americans—over ten times the number of WWI battle deaths. The tables below give the total numbers of Navy, Army, and civilian deaths due to the epidemic (based on data from A. W. Crosby, Jr., *Epidemic and Peace 1918*, Westport, Connecticut: Greenwood Press, 1976).

Week ending	Total Deaths Navy	Army	Total Civilian Deaths in 45 Major Cities
August 31	2		
September 7	13	40	
September 14	56	76	68
September 21	292	174	517
September 28	1172	1146	1970
October 5	1823	3590	6528
October 12	2338	9760	17,914
October 19	2670	15,319	37,853
October 26	2820	17,943	58,659
November 2	2919	19,126	73,477
November 9	2990	20,034	81,919
November 16	3047	20,553	86,957
November 23	3104	20,867	90,449
November 30	3137	21,184	93,641

(a) Find a logistic model to fit the set data for *Army* deaths. Do not round.

(b) Sketch the graph of the model, and mark where the curve is concave up and where it is concave down. Label the approximate location of the inflection point.

9. (6 pts.) Tell whether the given function is an increasing or decreasing exponential function, and give the constant percentage change.

$$f(x) = 72.378(0.695)^x$$

10. (6 pts.) (a) What is the limiting value of the following logistic function?

$$C(t) = \frac{3706.5}{1 + 8.976e^{-1.243x}}$$

(b) For the function $C(t)$ above, find $C(2.2)$ (do not round).

11. (6 pts.) You invest \$5000 at 3.8% APR compounded quarterly. Calculate the amount in the account after 7 years.

12. (6 pts.) You are saving for a down payment on a car. You have worked all summer and want to invest part of your earnings at 4.5% compounded monthly in order to make the down payment when you graduate. How much of your summer earnings should you invest now in order to have \$3000 in 2 years?

13. (10 pts.) The table below (based on *Statistical Abstract*, 1993) shows the population of the contiguous states of the United States.

Year	Population (millions)
1790	3.929
1810	7.240
1830	12.866
1850	23.192
1870	39.818
1890	62.948
1910	91.972
1930	122.775
1950	150.697
1970	202.229
1990	247.052

(a) Find an exponential model for the data. Do not round. Look at a scatter plot of the data and a graph of the model.

(b) Find a quadratic model for the data. Do not round. Look at a scatter plot of the data and a graph of the model.

(c) Which of the two models appears to have the better fit? Explain.