Tangents to Polynomials at Points on the y-axis

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(Based on exercises from "Elementary Functions," SMSG, Yale, 1961)

Problem

Given a polynomial,

(a) find the equation of the tangent to the polynomial at its point of intersection with the y-axis;

(b) graph the tangent and the polynomial on the same set of axes; and

(c) examine the tangent and polynomial graphs in a small neighborhood near the point of tangency.

Mathematical Background

To find the equation of the tangent to a polynomial graph at its point of intersection with the y-axis, simply omit every term from the polynomial whose degree is higher than one.

Maple Commands

To plot a function:

```
> plot(x^2+3*x,x=-3..3);
```

To specify the y-axis range:

```
> plot(x^2+3*x,x=-3..3,y=-20..0);
```

To plot two functions simultaneously:

```
> plot([x^2+3*x,x^3],x=-3..3);
```

Example

$y = 1 + x - 4\,x^2$

(a) Omit every term of degree higher than one. $y = 1 + x$

(b) Plot both:

```
> plot([1+x-4*x^2,1+x],x=-2..2);
```

(c) Use a small interval as the domain:

```
> plot([1+x-4*x^2,1+x],x=-0.1..0.1);
```

Use an even smaller interval:

```
> plot([1+x-4*x^2,1+x],x=-0.01..0.01);
```

Exercises

1. $y = 1 - x + x^2$

2. $y = 2 + 3\,x - 2\,x^2$

3. $y = 1 - x + x^3$

4. $y = 1 + 2\,x + x^5$

5. In your own words, describe what you observed in the graphs.