

Tangents to Polynomials at an Arbitrary Point p

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(Based on exercises from "Elementary Functions," SMSG, Yale, 1961)

Problem

Given a polynomial and a number p ,

- (a) find the equation of the tangent to the polynomial at $x=p$;
- (b) graph the tangent and the polynomial on the same set of axes.

Example

$$y = 1 + 2x - x^2 + 2x^3; \quad p = 4$$

- (a) As in the previous laboratory handout, rewrite the polynomial in powers of $(x-4)$:

$$1 + 2x - x^2 + 2x^3 = 121 + 90(x - 4) + 23(x - 4)^2 + 2(x - 4)^3.$$

Now omit all terms with powers of $(x-4)$ higher than the first:

$$\text{Answer: } 121 + 90(x - 4)$$

- (b) Plot both:

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> plot([1+2*x-x^2+2*x^3, 121+90*(x-4)], x=2..6);
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Exercises

1. $y = 3 + 4x + 2x^2 + x^3$; $p = 2$
2. $y = 4x^3 + x^2 + 3x$; $p = 3$
3. $y = 3 + 2x^3 + 4x^2$; $p = -3$
4. $y = 4x^3 - 3x^2 + 2x + 1$; $p = -4$