Rewriting Polynomials in Powers of (x-p)

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(Based on exercises from "Elementary Functions," SMSG, Yale, 1961)

Problem

Given a polynomial and a number p, use Maple to rewrite the polynomial in powers of (x-p). For example, $y = 1 + x - 4\,x^2$ is the same as $y = -32 - 23\,(x-3) - 4(x-3)^2$; in the second form it is written in powers of $(\,x-3)$.

Maple Commands

To expand a polynomial:

```
> expand(-32-23*(x-3)-4*(x-3)^2);
```

$$1 + x - 4\,x^2$$

Example

$y = 1 + 2\,x - x^2 + 2\,x^3; \quad p = 4;$

Begin with the highest power, and work down.

```
> expand(2*(x-4)^3);
```

$$2\,x^3 - 24\,x^2 + 96\,x - 128$$

Now $2\,x^3$ is ok, but we have $-24\,x^2$ instead of $-x^2$. Add $23(x-4)^2$ to correct it.

```
> expand(2*(x-4)^3+23*(x-4)^2);
```

$$2\,x^3 - x^2 - 88\,x + 240$$

Now $2\,x^3 - x^2$ is ok, but we have $-88\,x$ instead of $2\,x$. Add $90\,(x-4)$ to correct it.

```
> expand(2*(x-4)^3+23*(x-4)^2+90*(x-4));
```

$$2\,x^3 - x^2 + 2\,x - 120$$

All is ok except for the constant. Add 121 to correct it.

```
> expand(2*(x-4)^3+23*(x-4)^2+90*(x-4)+121);
```

$$2\,x^3 - x^2 + 2\,x + 1$$

Answer: $1 + 2\,x - x^2 + 2\,x^3 = 121 + 90(x-4) + 23(x-4)^2 + 2(x-4)^3$.

Exercises

1. $y = 3 + 4\,x + 2\,x^2 + x^3$; $p = 2$
2. $y = 4\,x^3 + x^2 + 3\,x$; $p = 3$
3. $y = 3 + 2\,x^3 + 4\,x^2$; $p = -3$
4. $y = 4\,x^3 - 3\,x^2 + 2\,x + 1$; $p = -4$