MTH 151
Exam 1
Spring 2025

Formulas are on the last page, which you may pull off.

Calculators are allowed. You may use a scientific calculator or a graphing calculator (e.g., TI-84) but not one with CAS (e.g., no TI-89, no TI-Nspire CAS). You may not use a phone app.

Show all work. Be neat and organized. Clearly indicate your answers.

100 points possible. 9 problems at 11 points each, plus 1 free point.

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1. Factor the expression by removing the common factor with the smaller exponent. Factor your answer completely.

$$3x^2(x-1)^4 - 2(x-1)^6$$

2. Use the table to evaluate each expression.

x	1	2	3	4	5	6
$f(x)$	5	3	6	2	4	1
$g(x)$	6	4	1	3	2	5

(a) $f(g(1))$

(b) $(f \circ g)(4)$

(c) $(g \circ f)(2)$

3. A student bought a smartwatch that tracks the number of steps she walks throughout the day. The table shows the number of steps recorded t minutes after 3:00 p.m. on the first day she wore the watch.

t (minutes)	0	10	20	30	40	50
Steps	3788	4914	5982	6901	7698	8124

(a) Find the slopes of the secant lines corresponding to the given intervals of t. This gives the student's average pace, in steps per minute, over the given interval.

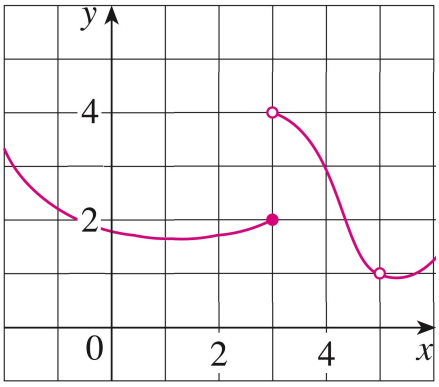
(i) $[0, 50]$

(ii) $[20, 30]$

(iii) $[30, 40]$

(b) Estimate the student's walking pace, in steps per minute, at 3:30 p.m. by averaging the slopes of two secant lines from part (a).

4. This figure shows the graph of $y = f(x)$.



For each of these, find the value or state that it does not exist.

(a) $\lim_{x \to 3^-} f(x)$

(b) $\lim_{x \to 3^+} f(x)$

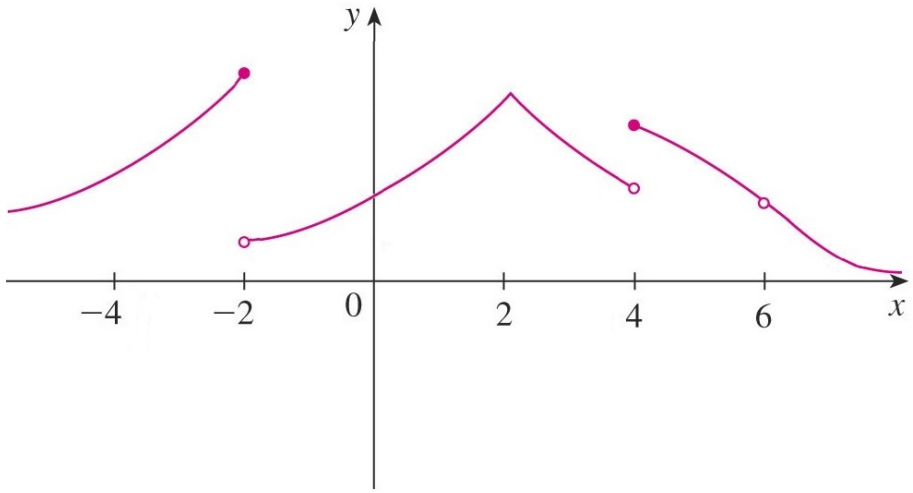
(c) $\lim_{x \to 3} f(x)$

(d) $f(3)$

5. Evaluate the limit, if it exists. Your answer must be fully supported with symbolic (algebraic) work.

$$\lim_{x \to 25} \frac{5 - \sqrt{x}}{25x^2 - x^3}$$

6. Use the graph to determine the x-values at which f is discontinuous. For each such x-value, determine whether f is continuous from the right, from the left, or neither.



7. Use the Intermediate Value Theorem to show that there is a solution of the equation in the given interval.

$$x^3 + 5x^2 - 15 = 0, \qquad (1, 2)$$

Solution

$f(x) = x^3 + 5x^2 - 15$ is _____ on the closed interval _____

$f(1) =$ _____

$f(2) =$ _____

Since _____ < _____ < _____,

there is a number c in $(1, 2)$ such that $f(c) =$ _____

by the Intermediate Value Theorem.

Thus, there is a _____ of the equation $x^3 + 5x^2 - 15 = 0$

in the interval $(1, 2)$.

8. Consider the following curve.

$$y = 4x^2 - 5x + 9$$

(a) Find the slope m of the tangent line at the point $(2, 15)$.

(b) Find an equation of the tangent line to the curve at the point $(2, 15)$.

9. Find the derivative of the function f using the definition of derivative.

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

In other words, do it "the long way." You can use the "four step process." Neatly show all important algebraic steps to justify your answer.

$$f(x) = \frac{1}{x-2}$$