

MTH 151
Exam 1
Spring 2014

1. Differentiate each function. You do not have to simplify your answers.

(a) (6 pts.) $y = \frac{x^2 + 4x + 3}{\sqrt{x}}$

(b) (6 pts.) $y = x^5 \sec x$

(c) (6 pts.) $y = \frac{2x + 5}{x^3 + 5}$

(d) (6 pts.) $y = \sqrt{x^2 + 16}$

(e) (6 pts.) $y = \tan^4(\sin x)$

2. (6 pts.) Evaluate the limit, if it exists. If the limit exists, your work must completely justify your answer algebraically. If the limit is ∞ or $-\infty$, say so, and support your answer with some reasoning.

$$\lim_{x \rightarrow -4} \frac{x^2 - 2x - 24}{x^2 + 11x + 28}$$

3. (6 pts.) Find the limit. Neatly show work that justifies the answer.

$$\lim_{x \rightarrow 0} \frac{\sin 4x}{x \cos 3x}$$

4. (12 pts.) Let

$$f(x) = \begin{cases} 7 - x, & x < 2 \\ -1, & x = 2 \\ x^2 + 2, & 2 < x < 4 \\ 3x, & x \geq 4. \end{cases}$$

State each of the following (or write DNE, if it does not exist).

(a) $\lim_{x \rightarrow 2^-} f(x)$

(b) $\lim_{x \rightarrow 2^+} f(x)$

(c) $\lim_{x \rightarrow 2} f(x)$

(d) $\lim_{x \rightarrow 4^-} f(x)$

5. (8 pts.) A tank holds 500 gallons of water, which drains from the bottom of the tank in half an hour. The values in the table show the volume $V(t)$ of water remaining in the tank (in gallons) after t minutes.

t (min)	5	10	15	20	25	30
$V(t)$ (gal)	347	222	125	56	14	0

(a) Find the average rate at which the volume V changed from $t = 5$ to $t = 30$. Include the units in your answer.

You do *not* have to write a sentence to interpret the meaning of your answer.

(b) Use the data in the table (without graphing) to estimate the rate at which the volume was changing at $t = 25$. Include the units in your answer.

You do *not* have to write a sentence to interpret the meaning of your answer.

6. (8 pts.) Evaluate the sine, cosine, and tangent of this angle. Give exact answers (with radicals if appropriate), not decimal approximations from your calculator.

$$\frac{7\pi}{6}$$

7. (8 pts.) Somebody used the product rule and got this. Simplify it.

(Just simplify. Do not take the derivative.)

$$\left(\frac{1}{2}\right)(4x+3)^{-1/2}(4)(x-5)^5 + (4x+3)^{1/2}(5)(x-5)^4$$

8. (10 pts.) A table of values for f , g , f' , and g' is given.

x	$f(x)$	$g(x)$	$f'(x)$	$g'(x)$
1	3	2	4	11
2	1	8	5	7
3	7	2	13	9

(a) If $h(x) = x^2g(x)$, find $h'(3)$.

(b) If $H(x) = g(f(x))$, find $H'(2)$.

9. (12 pts.) Find the derivative of the function f “the long way,” using the definition of derivative.

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

Neatly show all important algebraic steps to justify your answer.

$$f(x) = \sqrt{5 - 2x}$$

MTH 151
Exam 2
Spring 2014

1. (16 pts.) Find the indefinite integral. You do not have to simplify your answers.

(a) $\int (9x^3 + 2 \cos x) dx$

(b) $\int \frac{3x + 4x^2}{\sqrt{x}} dx$

2. (12 pts.) A particle is moving with the given data. Find the position function of the particle.

$$v(t) = 15t^2 - 4t, \quad s(1) = 17$$

3. (12 pts.) Find dy/dx by implicit differentiation.

$$5x^2y + 4y^3 = 50$$

4. (12 pts.) Use the Candidates Test (i.e., Closed Interval Method), showing all work, to find the absolute maximum and absolute minimum values of f on the given interval.

$$f(x) = 4x^3 + 15x^2 - 18x + 2, \quad [-4, 4]$$

5. (24 pts.) Given:

$$f(x) = \text{unknown continuous function, with domain the set of all real numbers}$$
$$f'(x) = \frac{6x - 4}{3x^{2/3}}, \quad f''(x) = \frac{6x + 8}{9x^{5/3}}$$

Use calculus methods, showing all work, to do the following.

(a) Find the intervals on which f is increasing or decreasing.

(b) Find the x -coordinates of all local maximum and local minimum points of f .

(You don't have to find the y -coordinates.)

(c) Find the intervals of concavity for f .

(d) Find the x -coordinates of all points of inflection of f .

(You don't have to find the y -coordinates.)

6. (12 pts.) A farmer wants to fence an area of 1210 square meters in a rectangular field and then divide it into 4 equal pens with some internal fences, all parallel to the same side of the rectangle. What dimensions of the field will minimize the total length of fencing? Use calculus methods. Include work that justifies why your answer gives an absolute minimum.

7. (12 pts.) Westville is 5 miles west of Eastport. Chris leaves Westville, traveling north at 3 mi/hr. At the same time, Pat leaves Eastport, traveling south at 4 mi/hr. After 2 hr, at what rate is the distance between Chris and Pat increasing?

(Suggestion: in your drawing, add auxiliary lines to obtain a single, larger right triangle.)