MTH 151
Exam 2
Spring 2010

Show all work in a neat and organized fashion. Clearly indicate your answers.
100 points possible.

1. (10 pts.) A particle moves in a straight line according to a law of motion

$$s = f(t) = 2t^3 - 21t^2 + 36t,$$

$t \geq 0$, where t is measured in seconds and s in feet.

(a) Find the velocity at time t.
(b) What is the velocity after 3 seconds?
(c) When is the particle at rest?
(d) When is the particle moving in the positive direction?
(e) Find the total distance traveled during the first 8 seconds.

2. (10 pts.) Find dy/dx by implicit differentiation.

$$y(x^2 + y) = x(y^2 - x)$$

3. (10 pts.) A street light is mounted at the top of a 16-ft-tall pole. A boy 4 ft tall walks away from the pole with a speed of 5 ft/sec along a straight path. How fast is the tip of his shadow moving when he is 45 ft from the pole?

4. (10 pts.) Use **differentials**, showing all work, to estimate the following.

$$\sqrt{64.18}$$

5. (10 pts.) Verify that the function satisfies the hypotheses of the Mean Value Theorem on the given interval $[a, b]$. Then find all numbers c such that

$$f'(c) = \frac{f(b) - f(a)}{b - a}.$$

$$f(x) = x^3 - 8x, \qquad [-5, 6]$$

6. (20 pts.) Find the derivative of each function.

(a) $y = \dfrac{\sin 8x}{x^3}$
(b) $y = x^2 \sin x + 4x \sec x$
(c) $y = \frac{1}{2} \cos x \sin 2x$
(d) $y = (5x - 2)^3 (8x^2 - 6x + 3)^{-5}$
(e) $y = \tan \sqrt{\sin x}$

7. (20 pts.) Given:

$f(x) =$ some continuous function, defined for all real numbers (no asymptotes)
$f'(x) = 7x(x - 4)^3 (x + 3)^2, \qquad f''(x) = 42(x - 4)^2 (x + 3)(x^2 - 2)$

Find the *exact* answers, with radical signs if necessary, not decimal approximations.

(a) Find the intervals on which f is increasing and those on which f is decreasing.
(b) Find the x-coordinates of all local maxima and local minima of f.
(c) Find the intervals of concavity.
(d) Find the x-coordinates of all points of inflection.

8. (10 pts.) The last page shows the graph of the first derivative f' of a function f.

(a) Find the intervals on which f is increasing and those on which f is decreasing.
(b) Find the x-coordinates of all local maxima and local minima of f.
(c) Find the intervals of concavity of f.
(d) Find the x-coordinates of all points of inflection of f.