

**MTH 151**  
**Exam 1**  
**Spring 2010**

Show all work in a neat and organized fashion. Clearly indicate your answers.  
100 points possible.

1. (10 pts.) Write equations for the tangent line and the normal line to the curve  $y = f(x)$  at the given point  $P$ .

$$y = 5x^2 + 4x - 6 \quad P(2, 22)$$

2. (10 pts.) The daily profit from the sale of  $x$  units of a product is given by  $P = 33x - 100 - 2x^2$  dollars.

- (a) What level of production maximizes profit?  
(b) What is the maximum possible profit?

3. (10 pts.) Evaluate this limit symbolically (algebraically), showing all significant algebraic steps.

$$\lim_{x \rightarrow 0} \frac{\sqrt{16 + x^2} - 4}{x^2}$$

4. (10 pts.) Suppose that  $f(x) = x^3 - 25x + 30$ , prove that there is at least one real number  $c$  such that  $f(c) = 0$ .

5. (10 pts.) On the attached page at the end is the graph of a function  $f$ . (Assume the axes have equal scales.) Sketch the graph of  $f'$  below it.

6. (10 pts.) Find the derivative of the function  $f$  using the definition of derivative.

$$f(x) = \frac{5x}{8 - 3x}, \quad f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

Show all important algebraic steps neatly to justify your answer.

7. (20 pts.) Differentiate each function. (You do not have to simplify your answers.)

(a)  $y = \frac{5x - 3}{4x + 11}$

(b)  $y = \frac{x^3}{\sqrt{x} + 25}$

(c)  $y = (x^5 + 1)(x^4 - 3)$

(d)  $y = 6\pi^2$

(e)  $y = \frac{x^5 - x\sqrt{x}}{x}$

8. (10 pts.) Prove the trig identity by transforming one side into the other. (Or, transform each side separately until you obtain the same expression.)

$$\csc x - \sin x = \cos x \cot x$$

9. (10 pts.) (a) Find the first three positive values of  $\theta$  (in degrees, to two decimal places) satisfying the given equation.

$$\sin \theta = -0.29$$

- (b) Find the first three positive values of  $x$  (in radians, to four decimal places) satisfying the given equation.

$$\cos x = -0.42$$