Math 151
Exam 3
Spring 2006

100 points possible.

1. (10 pts.) Let $f(x) = x^3 - 9x^2 - 48x + 5$. Find the absolute maximum and absolute minimum values of f on the interval $[-4, 3]$.

2. (10 pts.) Verify that the function satisfies the hypotheses of the Mean Value Theorem on the given interval $[a, b]$. Then find all numbers c such that

$$f'(c) = \frac{f(b) - f(a)}{b - a}.$$

$$f(x) = x^3 + 7x, \qquad [-6, 2]$$

3. (15 pts.) Given:

$f(x) =$ unknown continuous function, with domain the set of all real numbers

$f'(x) = (7x + 4)(x + 4)^5, \qquad f''(x) = 6(7x + 8)(x + 4)^4$

(a) Find the critical numbers of f.

(b) Find the intervals on which f is increasing or decreasing.

(c) Find the x-coordinates of all local maxima and local minima of f.

(d) Find the intervals of concavity.

(e) Find the x-coordinates of all points of inflection.

4. (10 pts.) Find the following limits exactly. Provide all major algebraic/symbolic steps to justify that your answer is correct.

(a)

$$\lim_{x \to +\infty} \frac{1 - 3x - 4x^2}{11x^2 - 7}$$

(b)

$$\lim_{x \to +\infty} \frac{12x^3 + 11}{\sqrt{7x^6 + 5x}}$$

5. (15 pts.) Given:

$$f(x) = 18x^4 - 134x^3 + 175x^2 + 632x + 852,$$
$$f'(x) = 72x^3 - 402x^2 + 350x + 632,$$
$$f''(x) = 216x^2 - 804x + 350.$$

Use a graphing calculator to estimate these answers to two decimal places.

(a) Find the critical numbers of f.

(b) Find the intervals on which f is increasing or decreasing.

(c) Find the x-coordinates of all local maxima and local minima of f.

6. (10 pts.) A piece of wire 20 meters long is cut into two pieces. One piece is bent into a square, and the other is bent into a rectangle whose length is twice its width. How should the wire be cut so that the total area enclosed is (a) a maximum? (b) a minimum?

7. (10 pts.) Use Newton's method to find the root of the given equation to the best accuracy your calculator will display. Use the specified initial approximation x_1, and show your values for x_2, x_3, x_4, and so on, until the algorithm stabilizes.

$$2x^3 + x^2 - x + 1 = 0, \qquad x_1 = -1.2$$

Recall that

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$$

and

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}.$$

8. (10 pts.) Find the most general antiderivative of the given function.

(a) $f(x) = \sec^2 x$

(b) $f(x) = \dfrac{25x^2 + 8}{x^5}$

9. (10 pts.) Find $f(x)$, given the following.

$$f''(x) = 36x^2 - 12x + 10, \qquad f(2) = 5, \quad f'(2) = 4$$