Math 151
Exam 2
Spring 2006

100 points possible.

1. Differentiate. (You do not have to simplify your answers.)

(a) (2 pts.) $y = x^2$

(b) (5 pts.) $y = x^5 - 4x^3 + x^{-2}$

(c) (5 pts.) $y = \dfrac{(x^3 + x)^4}{x^5 - 8}$

(d) (5 pts.) $y = \dfrac{1}{\sqrt{x^5 + 9}}$

(e) (5 pts.) $y = x^3 \sec x$

(f) (5 pts.) $y = \tan(\sin x)$

(g) (5 pts.) $y = (1 + \cos^3 x)^7$

2. (12 pts.) A particle moves according to a law of motion $s = f(t)$, $t \geq 0$, where t is measured in seconds and s in feet.

$$f(t) = t^3 - 15t^2 + 48t$$

(a) Find the velocity at time t.

(b) What is the velocity after 3 seconds?

(c) When is the particle at rest?

(d) When is the particle moving in the positive direction?

3. (12 pts.) Find the equation of the tangent line to the curve at the given point.

$$y = \frac{20x}{x^2 + 1}, \qquad (3, 6)$$

4. (12 pts.) Suppose f and g are the functions whose graphs are shown on the supplemental page.

(a) Let $H(x) = f(x)g(x)$. Find $H'(5)$, if it exists. If it does not exist, explain why.

(b) Let $u(x) = f(g(x))$. Find $u'(5)$, if it exists. If it does not exist, explain why.

5. (12 pts.) Let $y = (x^2 + 1)(x^3 + 1)$.

(a) Find the differential dy.

(b) Evaluate dy for $x = 2$ and $dx = 0.1$.

6. (10 pts.) Two cars start moving from the same point. One travels south at 50 mi/hr and the other travels west at 35 mi/hr. At what rate is the distance between the cars increasing 3 hours later?

7. (10 pts.) A student is sipping water from a conical paper cup whose axis is vertical. The height of the cup is 20 cm, and the diameter of the circular opening is 6 cm. How fast is the level of the water falling when the depth of the water is 5 cm, assuming that the student is drinking at a rate of $4\,\text{cm}^3/\text{sec}$?

(The volume of a cone is $V = \frac{1}{3}\pi r^2 h$.)