

Math 151
Exam 1
Spring 2006

100 points possible.

The figures for Problems 1, 2, and 3 are on a supplemental page.

1. (4 pts.) For the function g whose graph is given on the supplemental page, state the following.

$$\lim_{x \rightarrow 1^+} g(x)$$

- A. 1
- B. 2
- C. 2.6
- D. ∞
- E. Does not exist.

2. (4 pts.) The graph of a function f is given on the supplemental page. Select the true statement from the choices listed below the graph.

- A. $f(0) = 5$, $f'(0) = -1$, $f'(1) = 3$, $f'(2) = -5$
- B. $f(0) = 5$, $f'(0) = -1$, $f'(1) = -3$, $f'(2) = -5$
- C. $f(0) = 5$, $f'(0) = -1$, $f'(1) = -3$, $f'(2) = 5$
- D. $f(0) = 5$, $f'(0) = -1$, $f'(1) = 3$, $f'(2) = 5$
- E. $f(0) = 3$, $f'(0) = 1$, $f'(1) = 4$, $f'(2) = 6$

3. (4 pts.) Using the graph of h on the supplemental page, estimate the value of the derivative at the point $x = 0$.

- A. 0
- B. 4
- C. -4
- D. 2
- E. -2

4. (4 pts.) A rectangle has perimeter 10 m. Express the area of the rectangle as a function $A(l)$ of the length l of one of its sides.

- A. $A(l) = 5l + l^2$
- B. $A(l) = 10l - l^2$
- C. $A(l) = 5 - l$
- D. $A(l) = 10l + l^2$
- E. $A(l) = 5l - l^2$

5. (4 pts.) The monthly cost of driving a car depends on the number of miles driven. Lynn found that in October it cost her \$500 to drive 600 mi, and in July it cost her \$650 to drive 900 mi. Express the monthly cost C as a function of the distance driven d , assuming that a linear relationship gives a suitable model.

- A. $C = 0.5d - 200$
- B. $C = 200d + 0.5$
- C. $C = 200d - 0.5$
- D. $C = 0.5d + 200$
- E. $C = 0.5d - 100$

6. (4 pts.) Determine an appropriate viewing rectangle for the given function.

$$f(x) = \sin\left(\frac{x}{70}\right)$$

- A. $[0, 1]$ by $[0, 70]$
- B. $[-490, 490]$ by $[-1.5, 1.5]$
- C. $[-1, 0]$ by $[-70, 0]$
- D. $[-0.14, 0.14]$ by $[-1.5, 1.5]$
- E. $[-70, 70]$ by $[-70, 70]$

7. (4 pts.) If an arrow is shot upward on the moon with a velocity of 55 m/s, its height in meters after t seconds is given by $h = 55t - 0.04t^2$. Find the average velocity over the interval $[1, 1.04]$.

- A. 54.9194
- B. 55.0284
- C. 54.8174
- D. 54.9184
- E. 54.9084
- F. 54.9284

8. (4 pts.) Evaluate the limit, given that $\lim_{x \rightarrow 7} f(x) = -3$ and $\lim_{x \rightarrow 7} g(x) = 5$

$$\lim_{x \rightarrow 7} \frac{f(x)}{g(x)}$$

- A. 1
- B. $3/5$
- C. $-3/5$
- D. $-7/5$
- E. $-5/7$

9. (4 pts.) Choose an equation from the following that expresses the fact that a function f is continuous at the number 6.

A. $\lim_{x \rightarrow \infty} f(x) = 6$

B. $\lim_{x \rightarrow 6} f(x) = f(6)$

C. $\lim_{x \rightarrow \infty} f(x) = f(6)$

D. $\lim_{x \rightarrow 6} f(x) = 0$

E. $\lim_{x \rightarrow 6} f(x) = \infty$

10. (4 pts.) A curve has equation $y = h(x)$. Write an expression for the slope of the secant line through the points $P(8, h(8))$ and $Q(x, h(x))$.

11. (12 pts.) Let $f(x) = x^3 + 1$ and $g(x) = 1/x$.

(a) Find the function $f \circ g$ and simplify.

(b) Find the function $f \circ f$ and simplify.

12. (12 pts.) Evaluate the limit, if it exists. Your work must completely justify your answer algebraically. Please be neat.

$$\lim_{x \rightarrow 2} \frac{x^2 - x - 2}{x^2 - 4x + 4}$$

13. (12 pts.) Evaluate the limit, if it exists. Your work must completely justify your answer algebraically. Please be neat.

$$\lim_{h \rightarrow 0} \frac{\sqrt{2 + 5h} - \sqrt{2}}{h}$$

14. (12 pts.) Use the limit definition of derivative to show that the derivative of

$$f(x) = 3x^2 - 2x + 5$$

is $f'(x) = 6x - 2$. Your work must completely justify your answer algebraically. Please be neat.

15. (12 pts.) Use the limit definition of derivative to find the derivative of

$$f(x) = \frac{x}{3 - x}.$$

Your work must completely justify your answer algebraically. Please be neat.

Optional Bonus Problem. (8 optional bonus points possible) Suppose that $f(x) = x^3 + 3x - 2$, prove that there is at least one real number c such that $f(c) = 0$.