Math 151
Exam 2

Show all work in a neat and organized fashion. Clearly indicate your answers. 100 points possible.

1. (10 pts.) Find the derivative of each function. (You do not have to simplify your answer.)

(a) $f(x) = (3x - 8)^5(7x^2 - 3x + 1)^{14}$

(b) $g(x) = (1 + \sin^2 x)^7$

2. (10 pts.) Find dy/dx by implicit differentiation.

$$y^2 = x^2 + \sin(xy)$$

3. (10 pts.) A particle moves according to a law of motion

$$s = f(t) = t^3 - 3t^2 + 3t, \qquad (t \geq 0)$$

where t is measured in seconds and s in meters.

(a) Find the acceleration at time t, and after 2 seconds.

(b) Graph the position, velocity, and acceleration functions for $0 \leq t \leq 3$.

(c) When is the particle speeding up? When is it slowing down?

4. (10 pts.) A plane flying horizontally at an altitude of 1.5 miles and at a speed of 400 miles per hour passes directly over a radar station. Find the rate at which the distance from the plane to the station is increasing when it is 6 miles away from the station.

5. (10 pts.) Use **differentials**, showing all work, to estimate the following.

$$\frac{1}{50.1}$$

6. (10 pts.) Let $f(x) = 2x^3 - \frac{31}{2}x^2 + 5x$. Use calculus methods, showing all work, to do the following.

(a) Find the intervals of increase or decrease.

(b) Find the local maximum and minimum values.

$$f(x) = 2x^3 - \frac{31}{2}x^2 + 5x$$

7. (10 pts.) Let $f(x) = \frac{1}{12}x^4 - \frac{5}{2}x^2$. Use calculus methods, showing all work, to do the following.

Find the intervals of concavity and the inflection points.

$$f(x) = \frac{1}{12}x^4 - \frac{5}{2}x^2$$

8. (10 pts.) Find the dimensions of the rectangle of largest area that has its base on the x-axis and its other two vertices above the x-axis and lying on the parabola $y = 16 - x^2$.

9. (5 pts.) Use Newton's method with the specified initial approximation x_1 to find x_2, the second approximation to the root of the given equation.

$$2x^3 + x^2 - x + 1 = 0, \qquad x_1 = -1.2$$

Recall that

$$x_2 = x_n - \frac{f(x_1)}{f'(x_1)}$$

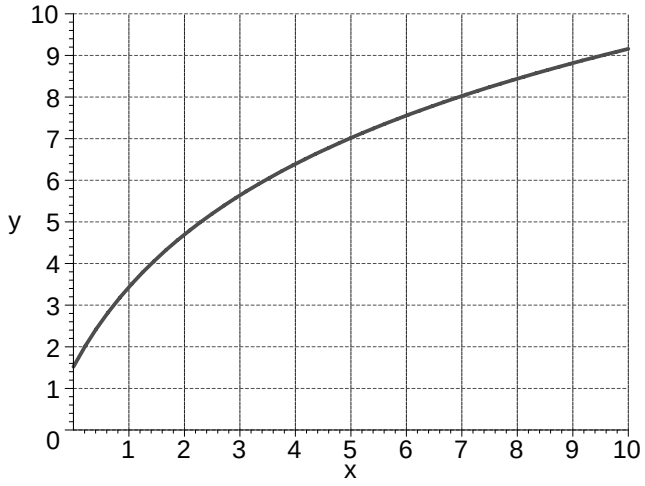
and

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}.$$

10. (5 pts.) Find $f(x)$, given the following.

$$f'(x) = x^3 + 5\sin x$$

11. (5 pts.) By reading values from the given graph of f, use five rectangles to find a lower estimate for the area under the given graph of f from $x = 0$ to $x = 10$. Sketch the rectangles that you use.



12. (5 pts.) The graph of f is shown. Evaluate the following integral by interpreting it in terms of areas.

$$\int_2^7 f(x)\,dx$$