Math 151
Quiz 5

Show all work in a neat and organized fashion. Clearly indicate your answers.
10 points possible.

No calculators on this quiz.

1. (4 pts.) A chemical reaction results in the formation of one or more substances (called *products*) from one or more starting materials (called *reactants*). For instance, the "equation"

$$2H_2 + O_2 \longrightarrow 2H_2O$$

indicates that two molecules of hydrogen and one molecule of oxygen form two molecules of water. Let's consider the chemical reaction

$$A + B \longrightarrow C$$

where A and B are the reactants and C is the product. The *concentration* of A is the number of moles (6.022×10^{23} molecules) per liter and is denoted by [A]. The concentration varies during a reaction, so [A], [B], and [C] are all functions of time t. The *rate of reaction* of C is $d[C]/dt$.

If one molecule of the product C is formed from one molecule of the reactant A and one molecule of the reactant B, and the initial concentrations of A and B have a common value $[A] = [B] = a$ moles/L, then it turns out that

$$[C] = a^2kt/(akt + 1)$$

where k is a constant.

(a) Find the rate of reaction of C at time t.

(b) Show that if $x = [C]$, then

$$\frac{dx}{dt} = k(a - x)^2.$$

2. (3 pts.) Differentiate $y = x^5 \tan x$. Do not simplify your answer.

3. (3 pts.) Differentiate $h(t) = \left(t^2 - \dfrac{1}{t^2}\right)^{5/2}$. Do not simplify your answer.

Optional Bonus Problem. (1 optional bonus point possible) If $\cot\theta = 3$ and $\pi < \theta < 2\pi$, find $\sin\theta$.