

Math 151**Exam 3**

Show all work in a neat and organized fashion. Clearly indicate your answers. You may use a TI-81/82/83, but you must permit me to clear the programs first. Some loaners are available. 100 points possible.

1. (4 pts.) Write the sum in expanded form.

$$\sum_{i=4}^6 3^i$$

2. (12 pts.) Find the general indefinite integral.

$$\int (x + 5)(4 - x^2) dx$$

3. (12 pts.) Find the general indefinite integral.

$$\int \left(\sqrt{x} - \frac{5}{\sqrt{x}} \right) dx$$

4. (12 pts.) Evaluate the definite integral.

$$\int_1^4 f(t) dt \quad \text{where } f(t) = \begin{cases} 3t, & \text{if } t < 3 \\ 5t^2, & \text{if } t \geq 3 \end{cases}$$

5. (12 pts.) Evaluate the limit symbolically (algebraically), showing all significant algebraic steps. Find the *exact* answer, with radical signs if necessary, not a decimal approximation.

$$\lim_{x \rightarrow +\infty} \frac{\sqrt{5 + 11x^2}}{3 + 7x}$$

6. (12 pts.) For the given function, find the intervals of concavity and the inflection points. (You only have to find the x -coordinates of the inflection points.) Find the *exact* answers, with radical signs if necessary, not decimal approximations.

$$f(x) = x^4 - 5x^2$$

7. (12 pts.) Use Newton's method $x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$ to find the positive solution of the equation given below, correct to as many decimal places as your calculator will display. Write down each x_1 , x_2 , x_3 , etc. from your work.

$$5 \sin x - x = 0$$

8. (12 pts.) **Set up** the max/min problem given below, but **do not solve it**.

(For your final answer, you must write down the function to be maximized or minimized, and the function must be in terms of one variable only. Also, say whether you are supposed to find the maximum or the minimum of this function.)

Find the point on the parabola $y = x^2 - 4x + 11$ that is closest to the origin.

9. (12 pts.) **Set up** the max/min problem given below, but **do not solve it**.

(For your final answer, you must write down the function to be maximized or minimized, and the function must be in terms of one variable only. Also, say whether you are supposed to find the maximum or the minimum of this function.)

A right circular cylinder is inscribed in a sphere of radius 27. Find the largest possible volume of such a cylinder. (Volume of a cylinder is $V = \pi r^2 h$.)

Optional Bonus Problem. (6 optional bonus points possible)

Find $\int_0^b x^3 dx$ the "hard way."

You must subdivide the interval $[0, b]$ into n subintervals, then find a Riemann sum. Then you have to simplify the Riemann sum so that you can take the limit as n approaches ∞ . Finally, take that limit. Upon request, I will provide any summation formulas that you need, without penalty.