Math 151
Exam 2

Show all work in a neat and organized fashion. Clearly indicate your answers.
100 points possible.

The following may or may not be useful.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

1. Differentiate each function. Do not simplify your answers.

(a) (4 pts.) $y = x^2$.

(b) (6 pts.) $f(x) = \dfrac{1}{\sqrt[3]{x}}$

(c) (6 pts.) $g(t) = (t^2 + 17)(t^3 - 3t + 1)$

(d) (6 pts.) $h(s) = \dfrac{2s^2 - 3s + 1}{2s + 1}$

(e) (6 pts.) $F(x) = \dfrac{x}{x + \dfrac{c}{x}}$

(f) (6 pts.) $H(x) = (3x - 2)^5(3 - x^2)^4$

(g) (6 pts.) $K(\theta) = \sec\theta \sin\theta$

(h) (6 pts.) $L(x) = \tan(\cos x)$

(i) (6 pts.) $k(x) = \sin^4(3x^2)$

2. (12 pts.) For the function given by $f(x) = x^3 - 5x^2 + x - 5$, find the intervals on which f is increasing and the intervals on which f is decreasing. (Find exact answers, not decimal approximations. Use calculus methods to justify your answers.)

3. (12 pts.) For the given function and the given interval $[a, b]$, find all numbers c in that interval for which $f'(c) = \dfrac{f(b) - f(a)}{b - a}$.

$$f(x) = \sqrt{x}, \qquad [1, 4]$$

4. (12 pts.) A rocket is launched vertically and is tracked by a radar station, which is located on the ground 3 miles from the launch site. What is the vertical speed of the rocket at the instant when its distance from the radar station is 5 miles and this distance is increasing at the rate of 5000 miles per hour?

5. (12 pts.) A water tank is in the shape of an inverted cone with vertex downward. The tank's radius is 3 ft and the tank is 5 ft high. At first the tank is full of water, but at time $t = 0$ (in seconds), a small hole at the vertex is opened, and the water begins to drain. When the height of water in the tank has dropped to 3 ft, the water is flowing out at 0.02 ft^3/sec. At what rate, in feet per second, is the water level dropping then? (The volume of a cone is $V = \frac{1}{3}\pi r^2 h$.)