Math 151
Old Exam 3

This was Exam 3, Fall 1998. It had 70 points over what is Section 4.7 to Section 6.1 in your book, and 30 points of "review" topics from material before that. (Exam 3 in Fall 1999 will not have "review" topics. It will have 100 points beginning with Section 4.7 and ending with a section yet to be announced.)

1. (10 pts.) A particle is moving with the given data. Find the position function $s(t)$ for the particle.

$$a(t) = -32, \qquad s(1) = 6, \qquad v(1) = 5$$

2. (10 pts.) Sketch the region bounded by the given curves and find the area of the region.

$$y = x^2 - 6x, \qquad y = 12x - 2x^2$$

3. (10 pts.) Let $f(x) = x^4 - 6x^2$. Find exact (not approximate) answers to the following questions. Use calculus to support your solutions.
(a) Find the critical numbers of f.
(b) Find the intervals on which f is increasing or decreasing.

4. (10 pts.) Find $\dfrac{dy}{dx}$.

$$x^2y^3 + 3y^2 = x - 4y$$

5. (10 pts.) A rectangular sheet of poster paper must have 1-inch margins at the bottom and sides and a 2-inch margin at the top. No printing will be permitted in the margins. The total area of the paper will be $200\,\text{in}^2$. What dimensions should the paper have in order to maximize the area available for printing? Use calculus methods.
Include a brief reason why you know your answer gives a maximum, not a minimum.

6. Find these integrals.

(a) (8 pts.) $\displaystyle\int_1^5 \frac{1}{x^3}\,dx$

(b) (8 pts.) $\displaystyle\int (x^3+1)^2\,dx$

(c) (8 pts.) $\displaystyle\int x^3 \sin(x^4+2)\,dx$

(d) (8 pts.) $\displaystyle\int x^3(1-x^2)^{3/2}\,dx$

7. (8 pts.) Use Part 1 of the Fundamental Theorem of Calculus to find the derivative of the given function.

$$H(x) = \int_5^{\sqrt{x}} t^7 \cos t\,dt$$

8. (10 pts.) Use the definition of derivative $f'(x) = \lim\limits_{h\to 0} \dfrac{f(x+h)-f(x)}{h}$ to find the derivative of the given function.

$$f(x) = \frac{1}{3x}$$

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