

Math 151
Old Exam 2

This was Exam 2, Fall 1998. It had 80 points over what is Section 3.7 to Section 4.5 in your book, and 20 points of “review” topics from material before that.

1. (10 pts.) Let $f(x) = 2x^3 + 3x^2 - 12x$. Find the absolute maximum and absolute minimum values of f on the interval $[-3, 4]$.

2. (10 pts.) For the given function $f(x)$ on the given interval $[a, b]$, find all numbers c in (a, b) such that $f'(c) = \frac{f(b) - f(a)}{b - a}$.

$$f(x) = 3x^2 + 6x - 5, \quad [-2, 1]$$

3. (20 pts.) Let $f(x) = x^4 - x^8$. Find exact (not approximate) answers to the following questions. Use calculus to support your solutions.

(a) Find the intervals on which f is increasing or decreasing.

(b) Find the x -coordinates of all local maxima and local minima of f .

(c) Find the intervals on which f is concave upward or concave downward.

(d) Find the x -coordinates of all points of inflection of f .

4. (15 pts.) Gravel is being dumped at a rate of $50 \text{ ft}^3/\text{min}$ onto a pile in the shape of a cone. The pile is forming so that the diameter of the base is always equal to the height. How fast is the height of the pile increasing when the pile is 15 ft high? (Volume of a cone: $V = \frac{1}{3}\pi r^2 h$.)

5. Find $\frac{dy}{dx}$. Please do *not* simplify your answers. Use parentheses carefully.

(a) (5 pts.) $y = \sin(x^5)$

(b) (5 pts.) $y = \sin^5 x$

(c) (5 pts.) $y = \frac{x^2 + 4}{x^5 + 3}$

(d) (10 pts.) $y^5 + 3x^2y^3 - 7x^6 = 8$

6. (5 pts.) Find the differential dy , if $y = \tan x$.

7. For each of the following limits that exists, find its value. (Show some work, but you don't have to quote limit laws.) If one doesn't exist, clearly explain why, and tell whether it can be expressed with the $+\infty$ or $-\infty$ symbol.

(a) (5 pts.) $\lim_{x \rightarrow 5} \frac{x - 5}{x^2 - 25}$

(b) (10 pts.) $\lim_{x \rightarrow +\infty} \left(\sqrt{x^2 + 3x + 1} - \sqrt{x^2 - 4x} \right)$