Math 151
Old Exam 1

This was Exam 1, Fall 1998. It was a week later, so it covered up through what is Section 3.6 in your book. It is a fairly decent sample exam, but replace Problem 5(e) with a different limit, and Problems 6(b,c,d,e) with problems from 3.3 #1–39 (especially 19–39). (This would give 35 points from Section 3.3. That section is very important, but I probably would not give more than 25 points from a single section.)

1. (10 pts.) Find an equation of the line through $(1, 5)$ and perpendicular to the line $7y - 9x + 2 = 0$.

2. (10 pts.) An open rectangular box (with a bottom but no top) with volume 50 cubic meters has a square base. Express the surface area of the box as a function of the length of a side of the base.

3. (10 pts.) Find all points of discontinuity for the function $f(x) = \dfrac{3x^2 + 3x - 60}{x^2 - 6x + 8}$. For each such point c, tell whether the value $f(c)$ can be redefined in order to make f continuous at c.

4. (10 pts.) Find the values of A, B, and C so that the parabola $y = Ax^2 + Bx + C$ will pass through the point $(2, 11)$, and so that the line $y = 10x - 12$ is tangent to the parabola at the point $(3, 18)$.

5. (25 pts.) Instructions: For each of the following limits that exists, find its value. (Show some work, but you don't have to quote limit laws.) If one doesn't exist, clearly explain why, and tell whether it can be expressed with the $+\infty$ or $-\infty$ symbol.

(a) $\lim\limits_{x\to 5} \dfrac{x - 5}{x^2 - 25}$

(b) $\lim\limits_{t\to 0} \dfrac{\sqrt{t^2 + 16} - 4}{t^2}$

(c) $\lim\limits_{x\to 4^+} \dfrac{5}{4 - x}$

(d) $\lim\limits_{x\to 1} f(x)$, where $f(x) = \begin{cases} x^2 - 4x + 3, & x < 1 \\ x - 3, & x \geq 1. \end{cases}$

(e) $\lim\limits_{t\to 0} \dfrac{\cos 3t}{\cos 2t}$

6. (25 pts.) Find $\dfrac{dy}{dx}$. Please do *not* simplify your answers. Use parentheses carefully.

(a) $y = \dfrac{x^2 + 4x + 3}{\sqrt{x}}$

(b) $y = \dfrac{\sin x}{1 + \cos x}$

(c) $y = \sin \dfrac{1}{x}$

(d) $y = \tan(x^2) + \tan^2 x$

(e) $y = \sqrt[4]{\dfrac{x^3 + 1}{x^3 - 1}}$

7. (10 pts.) Use the definition of derivative $f'(x) = \lim\limits_{h\to 0} \dfrac{f(x + h) - f(x)}{h}$ to find the derivative of $f(x) = \dfrac{4}{2 - 5x}$.

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