MTH 151
Exam 1, Form A
Fall 2022

Show all work in a neat and organized fashion. Clearly indicate your answers.
100 points possible.

Graphing calculator OK but not one with CAS (e.g., no TI-89, no TInspire).

These formulas may or may not be useful.

$$a^3 + b^3 = (a+b)(a^2 - ab + b^2) \qquad a^3 - b^3 = (a-b)(a^2 + ab + b^2)$$

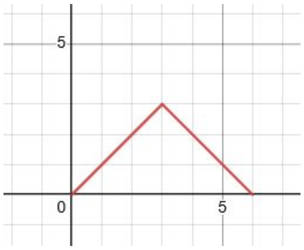
$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

0. (1 pt.) One free point.

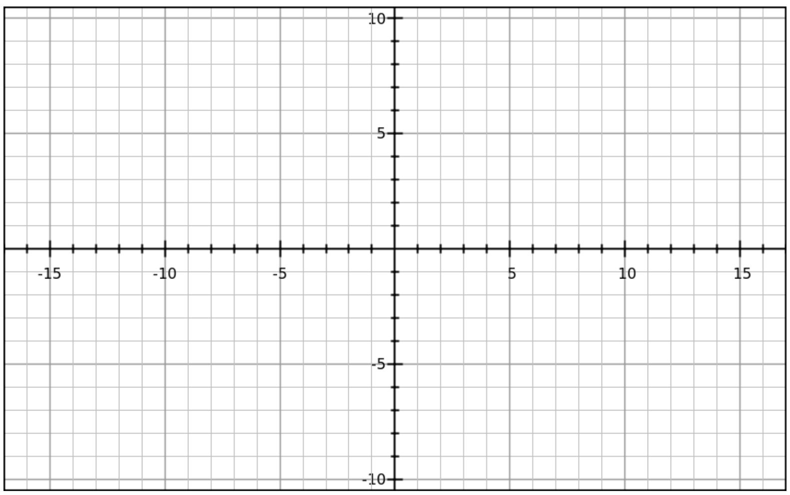
1. (11 pts.) Completely factor the expression.

$$6(5x+2)^2(3-x)^2 + (5x+2)(3-x)^3$$

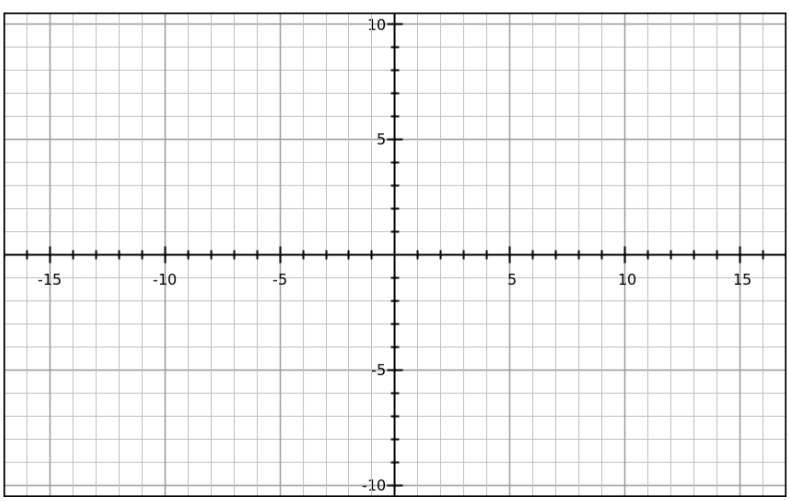
2. (11 pts.) The graph of f is given. Use it to graph the following functions.



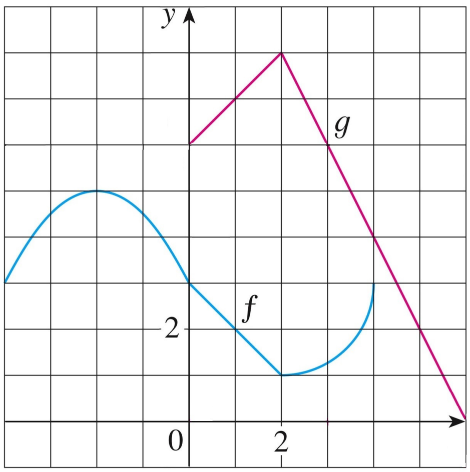
(a) $y = f(3x)$



(b) $y = f(-x)$



3. (11 pts.) Consider the given graphs of f and g.



Evaluate each expression, or explain why it is undefined.

(a) $f(g(5))$

(b) $(f \circ g)(4)$

(c) $(g \circ f)(2)$

(d) $(g \circ g)(5)$

(e) $(f \circ f)(1)$

4. (11 pts.) Researchers measured the average blood alcohol concentration $C(t)$ of eight men starting one hour after consumption of 30 mL of ethanol (corresponding to two alcoholic drinks).

t (hours)	1.0	1.5	2.0	2.5	3.0	3.5
$C(t)$ (g/dL)	0.043	0.032	0.024	0.016	0.007	0.003

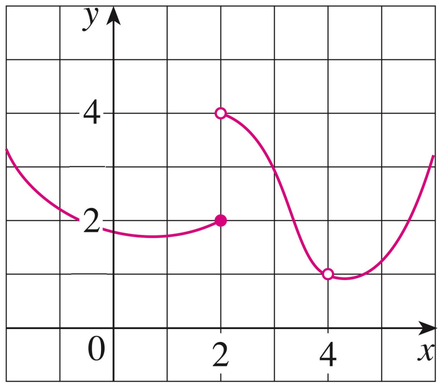
(a) Find the average rate of change of C with respect to t over the time interval $[1.0, 2.5]$. Show your work. Include the units in your answer.

You do *not* have to write a sentence to interpret the meaning of your answer.

(b) Use the data in the table (without graphing) to estimate the rate at which C was changing at $t = 2.5$. Show your work. Include the units in your answer.

You do *not* have to write a sentence to interpret the meaning of your answer.

5. (11 pts.) This figure shows the graph of $y = f(x)$.



For each of these, find the value or state that it does not exist.

(a) $\lim_{x \to 2^-} f(x)$

(b) $\lim_{x \to 2^+} f(x)$

(c) $\lim_{x \to 4^-} f(x)$

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6. (11 pts.) Evaluate the limit, if it exists. Your answer must be fully supported with symbolic (algebraic) work.

$$\lim_{x \to 0} \frac{\sqrt{36 + x^2} - \sqrt{36 - x^2}}{4x^2}$$

7. (11 pts.) (a) Explain why the function f is discontinuous at the given number a.

$$f(x) = \begin{cases} 5 - x & \text{if } x < 1 \\ x^2 & \text{if } x \geq 1 \end{cases} \qquad a = 1$$

(b) Sketch the graph of f.

8. (11 pts.) As a reminder, this is the Intermediate Value Theorem.

Intermediate Value Theorem. Suppose
(1) f is continuous on the closed interval $[a, b]$, and
(2) N is any number strictly between $f(a)$ and $f(b)$.
Then there exists at least one number c in (a, b) such that $f(c) = N$.

Use the Intermediate Value Theorem to show that there is a solution of the given equation in the specified interval.

$$-x^3 + 6x + 2 = 0, \qquad (-1, 2)$$

9. (11 pts.) Let $W(t)$ be the number of words per minute (wpm) that a student in a typing class can type after t weeks in the course.

In parts (a), (b), and (c), interpret each statement. Select the best answer from the answer bank below.

(a) $W(30) = 25$

(b) $W'(30) = 25$

(c) $\dfrac{dW}{dt} = 30$ when $t = 25$

Answer bank

A. During the first 30 weeks of the course, the student's typing speed increased at a rate of 25 wpm per week, on average.

B. When the student was 30 weeks into the course, the student was typing 25 wpm.

C. When the student was typing 30 wpm, the student was 25 weeks into the course.

D. When the student was 30 weeks into the course, the student's typing speed was increasing at a rate of 25 wpm per week.

E. When the student's typing speed was 30 wpm, the number of weeks into the course was increasing at a rate of 25 weeks per wpm.

F. When the student was 25 weeks into the course, the student's typing speed was increasing at a rate of 30 wpm per week.

G. During the first 25 weeks of the course, the student's typing speed increased at a rate of 30 wpm per week, on average.

MTH 151
Exam 1, Form B
Fall 2022

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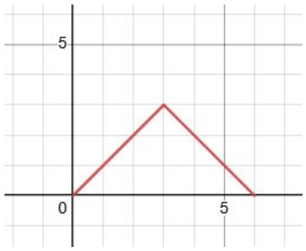

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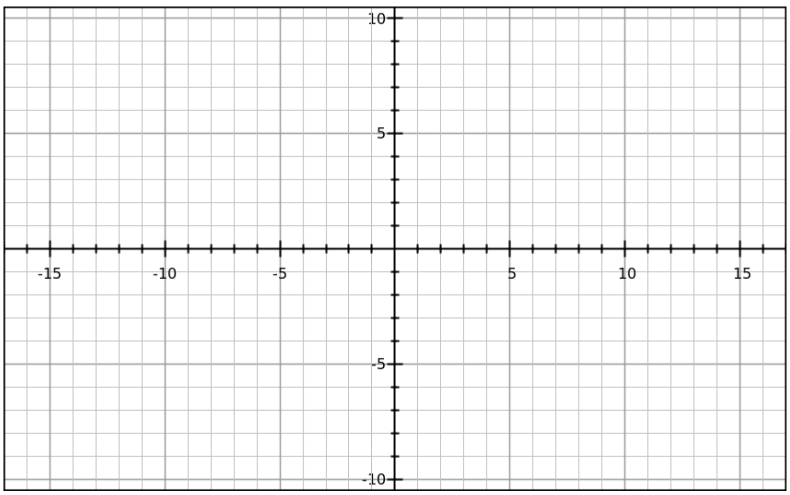

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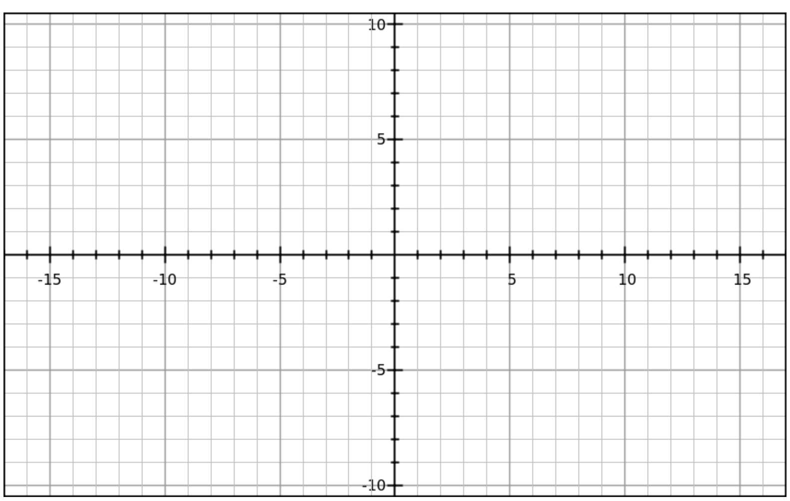

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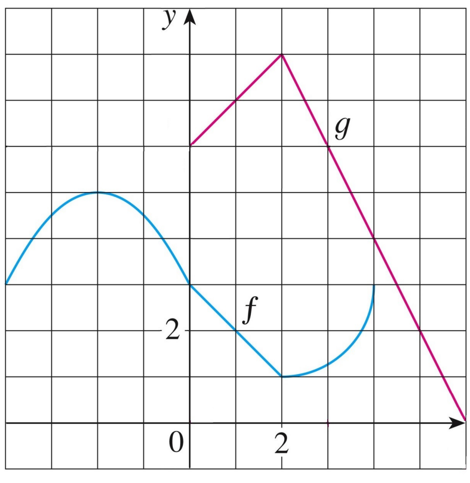

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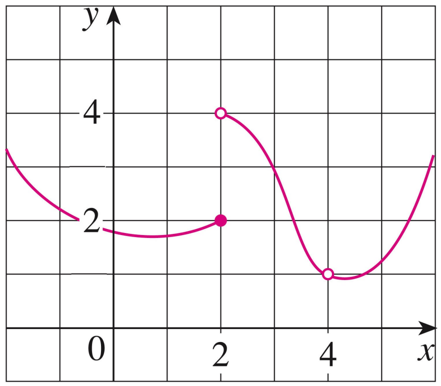

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