MTH 151
Exam 3
Fall 2013

Show all work in a neat and organized fashion. Clearly indicate your answers.
100 points possible.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

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1. (25 pts.) Given:

$$f(x) = 2x^3 + 15x^2 - 36x$$

Use calculus methods, showing all work, to do the following.

(a) Find the intervals on which f is increasing or decreasing.

(b) Find the x-coordinates of all local maximum and local minimum points of f.
(You don't have to find the y-coordinates.)

(c) Find the intervals of concavity for f.

(d) Find the x-coordinates of all points of inflection of f.
(You don't have to find the y-coordinates.)

2. (25 pts.) Given:

$f(x) =$ unknown continuous function, with domain the set of all real numbers

$$f'(x) = \frac{4x+5}{3x^{2/3}}, \qquad f''(x) = \frac{4x-10}{9x^{5/3}}$$

Use calculus methods, showing all work, to do the following.

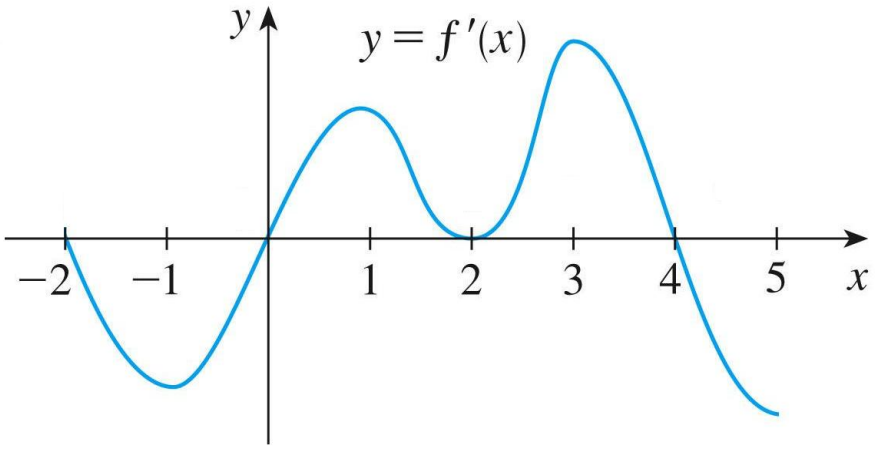
(a) Find the intervals on which f is increasing or decreasing.

(b) Find the x-coordinates of all local maximum and local minimum points of f.
(You don't have to find the y-coordinates.)

(c) Find the intervals of concavity for f.

(d) Find the x-coordinates of all points of inflection of f.
(You don't have to find the y-coordinates.)

3. (20 pts.) The graph of the *derivative* f' of a function f is shown.



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4. (15 pts.) Verify that the function satisfies the hypotheses of the Mean Value Theorem on the given interval $[a, b]$. Then find all numbers c in (a, b) such that

$$f'(c) = \frac{f(b) - f(a)}{b - a}.$$

$$f(x) = x^{2/3}, \qquad [0, 1]$$

5. (15 pts.) Set up the following applied problem on a closed interval and use the Candidates Test (i.e., Closed Interval Method), showing all work, to justify your solution.

A rectangular box has a square base. The edge of the base must be at least 1 ft. The box has no top, and the total area of its five sides is 192 ft^2. What is the maximum possible volume of such a box?

Optional Bonus Problem. (5 optional bonus points possible) Find the limit. Support your answer with symbolic/algebraic work.

$$\lim_{x \to \infty} \frac{5x^3 - 6x^2 + 3}{2x^3 + 5x - 4}$$