MTH 151
Exam 3
Fall 2011

Show all work in a neat and organized fashion. Clearly indicate your answers.
100 points possible.

$$\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$$

$$\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$$

$$\sum_{i=1}^{n} i^3 = \left[\frac{n(n+1)}{2}\right]^2$$

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1. (10 pts.) Find the limit or show that it does not exist. (Support your answer with symbolic work.)

$$\lim_{x\to\infty} \frac{5x^2 - 9x + 3}{11x^2 + 4x - 8}$$

2. (15 pts.) Given:

$$\begin{aligned} f(x) &= \text{ some unknown continuous function with domain all of } \mathbb{R} \\ f'(x) &= x^3(x-2)^2(7x-8) \\ f''(x) &= 6x^2(x-2)(7x^2-16x+8) \end{aligned}$$

(a) Find the intervals on which f is increasing or decreasing.

(b) Find the x-coordinates of all local maxima and local minima of f. For each answer, how do you know it is a local maximum/minimum?

3. (15 pts.) The graph of the second derivative f'' of a function f is shown on the attached page. The domain of f is $[-1, 7]$, and f'' exists on $(-1, 7)$.

(a) Find the intervals of concavity of f.

(b) Find the x-coordinates of all points of inflection of f. For each answer, how do you know it is an inflection point?

4. (10 pts.) A rectangular storage container with a square base and a square top is made to contain $1000\,\text{ft}^3$ of liquid. Material for each side is very expensive; it costs \$216 per square foot. Material for the base and for the top costs only \$8 per square foot.

Find a formula for the total cost of the container. Then find the dimensions of the square base that gives minimum total cost. Justify why your answer gives an absolute minimum.

5. (10 pts.) Use Newton's method to find the root of the given equation to the best accuracy your calculator will display. Use the specified initial approximation x_1, and show your values for x_2, x_3, x_4, and so on, until the algorithm stabilizes.

$$x^3 - x^2 - 2 = 0, \qquad x_1 = 2$$

Recall that

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)} \qquad \text{and} \qquad x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}.$$

6. (20 pts.) Find the indefinite integral. You do not have to simplify your answers.

(a) $\displaystyle\int (4x^3 + \sec^2 x)\, dx$

(b) $\displaystyle\int \frac{6}{x^3}\, dx$

(c) $\displaystyle\int x^3 \sqrt{x^4 + 1}\, dx$

(d) $\displaystyle\int x^2 \cos(2x^3)\, dx$

7. (10 pts.) A particle is moving with the given data. Find the position function of the particle.

$$v(t) = 6t + 12t^2, \qquad s(1) = 15$$

8. (10 pts.) Find the limit.

$$\lim_{n\to\infty} \sum_{i=1}^{n} \frac{5}{n} \left(\frac{2i}{n}\right)^2$$