MTH 151
Exam 1
Fall 2011

Show all work in a neat and organized fashion. Clearly indicate your answers.
100 points possible.

$$\cos(s+t) = \cos s \cos t - \sin s \sin t$$
$$\cos(s-t) = \cos s \cos t + \sin s \sin t$$
$$\sin(s+t) = \sin s \cos t + \cos s \sin t$$
$$\sin(s-t) = \sin s \cos t - \cos s \sin t$$

1. (7 pts.) Find the domain of the function.

$$f(x) = \frac{1}{\sqrt{1-4x}}$$

2. (6 pts.) Classify each function as a power function, root function, polynomial (state its degree), rational function, algebraic function, trigonometric function, exponential function, or logarithmic function.

(a) $h(x) = \dfrac{2x^3}{1-x^2}$

(b) $u(t) = 1 - 1.1t + 2.54t^2$

(c) $v(t) = 5^t$

3. (7 pts.) A weight attached to a vertical spring is bouncing up and down. Its distance from the floor is given by a transformed sine or cosine graph. A stopwatch is started at time $t = 0$ when the weight is at its high point, 80 cm above the floor. At a time 1.6 seconds later, the weight reaches its next low point, 50 cm above the floor. Find a function that models the distance from the floor to the weight as a function of time.

4. (7 pts.) On an attached page, the graph of h is given. Use it to graph the following function.

$$y = h(\tfrac{1}{2}x)$$

You may draw your graph here or on the attached page.

5. (7 pts.) Given $F(x) = \cos(\sin^2 x)$, find (nontrivial) functions f, g, and h such that $F = f \circ g \circ h$.

6. (7 pts.) A tank holds 500 gallons of water, which drains from the bottom of the tank in half an hour. The values in the table show the volume $V(t)$ of water remaining in the tank (in gallons) after t minutes.

t (min)	5	10	15	20	25	30
$V(t)$ (gal)	347	222	125	56	14	0

(a) Find the average rate at which the volume V changed from $t = 10$ to $t = 30$. Include the units in your answer.

(b) Use the data in the table (without graphing) to estimate the rate at which the volume was changing at $t = 17.5$. Include the units in your answer.

7. (12 pts.) On the attached page, the graph of a function g is given. State each of the following (or write DNE, if appropriate).

(a) $\lim_{x \to 2^-} g(x)$

(b) $\lim_{x \to -4^+} g(x)$

(c) $g(2)$

(d) $\lim_{x \to 7^+} g(x)$

(e) $\lim_{x \to 7^-} g(x)$

(f) The equation(s) of the vertical asymptote(s).

8. (7 pts.) Let $g(x) = \dfrac{|x-1|}{x^2+5x-6}$.

(a) Find $\lim\limits_{x\to 1^+} g(x)$.

(b) Find $\lim\limits_{x\to 1^-} g(x)$.

(c) Does $\lim\limits_{x\to 1} g(x)$ exist?

9. (7 pts.) Is f continuous at a? Show the work that leads to your answer, using the mathematical definition of continuous.

$$f(x) = \begin{cases} 15 - x^2, & \text{if } x < 3 \\ 4x - 6, & \text{if } x \geq 3 \end{cases} \qquad a = 3$$

10. (7 pts.) Prove that there is a root of the given equation in the specified interval.

$$x^3 + 5x - 8 = 0 \qquad (1, 2)$$

11. (7 pts.) If $\sec x = \frac{5}{3}$ and $\sin y = \frac{1}{4}$, where x and y lie between 0 and $\pi/2$, evaluate the expression.

$$\cos(x + y)$$

12. (6 pts.) The limit represents the derivative of some function f at some number a. State such an f and a in each case.

(a) $\lim_{x \to 4} \dfrac{3^x - 81}{x - 4}$

(b) $\lim_{h \to 0} \dfrac{(5+h)^3 - 125}{h}$

13. (6 pts.) Use the given graph on the attached page to estimate the value of each derivative.

(a) $f'(0)$

(b) $f'(2)$

14. (7 pts.) Find the derivative of the function f using the definition of derivative.

$$f(x) = \sqrt{7 - x}, \qquad f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

Show important algebraic steps neatly to justify your answer. Hopefully you will find that $f'(x) = \dfrac{-1}{2\sqrt{7-x}}$. If your answer doesn't work out, don't fake it (because that would be an additional error); just do your best.