Math 151
Exam 2
Fall 2008

Justify all answers with neat and organized work. Clearly indicate your answers. 100 points possible.

1. (20 pts.) Find the differential dy (you do not have to simplify your answers).

(a) $y = (x^6 + 3x)^4$

(b) $y = \sqrt{x^5 + 2x}$

(c) $y = \dfrac{4x + 1}{x^5 - 9}$

(d) $y = 4\cos^5 x$

(e) $y = x^5 \tan x$

2. (10 pts.) Find dy/dx by implicit differentiation.

$$x^2y + 3xy^3 - x = 3$$

3. (10 pts.) Find the absolute maximum and absolute minimum values of f on the given interval. Use calculus methods, and show all work!

$f(x) = 2x^3 - 3x^2 - 12x, \qquad [-2, 3]$

4. (10 pts.) Let $f(x) = x^4 - 2x^2 + 7$. Use calculus methods, showing all work, to do the following.

(a) Find the intervals of increase or decrease.

(b) Find the local maximum and minimum values. (Find both the x and y values.)

5. (10 pts.) The derivative of a function f is $f'(x) = (8x + 5)^3(x - 3)^2(x - 9)^5$.

(a) Find the intervals of increase or decrease for f.

(b) Find the x-coordinates of all local maxima and local minima of f.

6. (10 pts.) A particle moves in a straight line according to a law of motion

$$s = f(t) = t^3 - 6t^2 + 9t + 1,$$

$t \geq 0$, where t is measured in seconds and s in feet.

(a) Find the velocity at time t.

(b) What is the velocity after 3 seconds?

(c) When is the particle at rest?

(d) Find the acceleration at time t.

(e) When is the particle speeding up?

7. (10 pts.) Use differentials to estimate $\sqrt{64.04}$ (your work must clearly show that you know how to use differentials to find this estimate).

8. (10 pts.) Recall the following theorem.

Mean Value Theorem. *Suppose*

(1) *f is continuous on the closed interval $[a, b]$; and*
(2) *$f'(x)$ exists for all x such that $a < x < b$.*

Then there must exist at least one number c with $a < c < b$ such that

$$f'(c) = \frac{f(b) - f(a)}{b - a}.$$

Verify that the following function satisfies the two hypotheses of the Mean Value Theorem on the given interval. Then find all numbers c that satisfy the conclusion of the Mean Value Theorem.

$$f(x) = x^3 + 5x + 7, \qquad [1, 3]$$

9. (10 pts.) Westville is 20 miles due west of Eastville. At noon, a car leaves Westville, moving north at 60 mi/hr. At 2:00 pm, another car leaves Eastville, moving east at 45 mi/hr. At what rate is the distance between the cars increasing at 3:00 pm?