Math 151
Exam 1
Fall 2008

Justify all answers with neat and organized work. Clearly indicate your answers. 100 points possible.

1. (10 pts.) Find the equation of the line through $(1, 5)$ and parallel to the line $3x + 4y = 24$.

2. (10 pts.) Find the equation for the tangent line and the normal line to the curve $y = 20 - 8x - 3x^2$ at the point where $x = 2$.

3. (10 pts.) The sensitivity S to a drug is related to the dosage x in milligrams by $S = 800x - 3x^2$.

(a) What dosage gives maximum sensitivity?

(b) Determine the maximum sensitivity.

4. (10 pts.) For the function g whose graph is given on the supplemental page, state the value of each quantity, if it exists. If it does not exist, explain why.

(a) $\lim_{x \to 1^-} g(x)$

(b) $\lim_{x \to 1^+} g(x)$

(c) $\lim_{x \to 1} g(x)$

(d) Find $\lim_{x \to 4} [x^3 g(x)]$

5. (10 pts.) In parts (a) and (b), suppose that a function f is continuous on $[2, 7]$ except at 4 and that $f(2) = 8$ and $f(7) = 1$. Let $N = 6$.

(a) Sketch a possible graph of f, showing that f might not satisfy the conclusion of the Intermediate Value Theorem.

(b) Sketch a possible graph of f, showing that f might still satisfy the conclusion of the Intermediate Value Theorem (even though it doesn't satisfy the hypothesis).

6. (10 pts.) Use the "four-step process" to find the derivative $f'(x)$, given

$$f(x) = \frac{4}{3-2x}.$$

In other words, evaluate the following limit symbolically (algebraically), showing all significant algebraic steps.

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

7. (10 pts.) Differentiate each function. (You do not have to simplify your answers.)

(a) $A(x) = \dfrac{8}{x^{10}}$

(b) $u = \sqrt[5]{t}$

(c) $y = \dfrac{t^2 + 9}{t^5 - 4t^2 + 3}$

8. (10 pts.) Given: $f(2) = 4$, $f'(2) = 5$, $g(2) = 6$, and $g'(2) = 7$.

(a) Let $h(x) = 3x^4 f(x)$. Find $h'(2)$.

(b) Let $k(x) = \dfrac{f(x)}{x^3 g(x)}$. Find $k'(2)$.

9. (10 pts.) Find the remaining trigonometric ratios exactly. (Do not find decimal approximations.)

$$\sin\theta = -\frac{4}{7}, \qquad \pi < \theta < \frac{3\pi}{2}$$

10. (10 pts.) Differentiate each function. (You do not have to simplify your answers.)

(a) $\cos(x^2 \tan x)$

(b) $\left(\dfrac{\sin x}{x^4 + 3x + 5}\right)^4$

(c) $\sec^3(\sin(x^5))$