MTH 110
Quiz 2
Spring 2018

Show all work in a neat and organized fashion. Clearly indicate your answers.
20 points possible. You may use a calculator. No phones.

The following formulas may or may not be useful.

$$C = \frac{n(n-1)}{2}$$

$$\text{Standard divisor} = \frac{\text{total population}}{\text{total number of seats}}$$

$$\text{Standard quota for a state} = \frac{\text{population of that state}}{\text{standard divisor}}$$

$$\text{Percent increase} = \frac{\text{amount of increase}}{\text{original amount}}$$

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1. (4 pts.) The preference table for an election is shown.

Number of Votes	9	9	9	9	3	3	3	3
1st choice	E	A	E	A	B	B	D	A
2nd choice	C	E	B	E	D	A	C	E
3rd choice	D	C	C	B	A	E	E	D
4th choice	B	D	D	C	C	C	B	B
5th choice	A	B	A	D	E	D	A	C

Suppose the pairwise comparison method is used. Who is the winner? (Show your work, showing all comparisons.)

2. (6 pts.) The preference table gives the results of a straw vote among three candidates A, B, and C.

Number of Votes	40	32	28	16
1st choice	B	A	C	C
2nd choice	C	B	A	B
3rd choice	A	C	B	A

(a) Suppose the plurality-with-elimination method is used. Who is the winner of the straw vote? (Show all work.)

(b) In the actual election, the 16 voters in the last column who voted CBA, in that order, change their votes to BCA. Using the plurality-with-elimination method, who wins the actual election? (Show all work.)

(c) Is the monotonicity criterion satisfied? Using one or more complete sentences, explain your answer.

3. (6 pts.) A country has two states, A and B. The congress has 100 seats, divided between the two states according to their respective populations. The table shows the population of each state, in thousands.

State	A	B	Total
Population (in thousands)	2090	17,910	20,000

(a) Use Hamilton's method to apportion the seats.

(b) Suppose that a third state, C, with the population shown in the table, is added to the country.

State	A	B	C	Total
Population (in thousands)	2090	17,910	1050	21,050

The country adds 5 new seats for state C. Use Hamilton's method to reapportion the 105 seats.

(c) What paradox occurs? Explain briefly.

4. (2 pts.) An ice cream store sells two drinks (sodas or milk shakes), in four sizes (mini, small, medium, or large), and three flavors (strawbery, vanilla, or chocolate). In how many ways can a customer order a drink?

5. (2 pts.) You are taking a multiple-choice test that has six questions. Each of the questions has four answer choices, with one correct answer per question. If you select one of these four choices for each question and leave nothing blank, in how many ways can you answer the questions?

Voting Method	How the Winning Candidate Is Determined
Plurality Method	The candidate with the most first-place votes is the winner.
Bourda Count Method	Voters rank all candidates from the most favorable to the least favorable. Each last-place vote receives 1 point, each next-to-last-place vote 2 points, and so on. The candidate with the most points is the winner.
Plurality-with-Elimination Method	The candidate with the majority (over 50%) of first-place votes is the winner. If no candidate receives a majority, eliminate the candidate with the fewest first-place votes. Either hold another election or adjust the preference table. Continue this process until a candidate receives a majority of first-place votes. That candidate is the winner.
Pairwise Comparison Method	Voters rank all the candidates. A series of comparisons is made in which each candidate is compared to each of the other candidates. The preferred candidate in each comparison receives 1 point; in case of a tie, each receives $\frac{1}{2}$ point. The candidate with the most points is the winner.

Fairness Criterion	Description
Majority Criterion	If a candidate receives a majority of first-place votes in an election, then that candidate should win the election.
Head-to-Head Criterion	If a candidate is favored when compared head-to-head with every other candidate, then that candidate should win the election.
Monotonicity Criterion	If a candidate wins an election and, in a reelection, the only changes are changes that favor the candidate, then that candidate should win the reelection.
Irrelevant Alternatives Criterion	If a candidate wins an election and, in a recount, the only changes are that one or more of the other candidates are removed from the ballot, then that candidate should still win the election.

Method	**Divisor**	**Apportionment**
Hamilton's	Standard divisor $= \dfrac{\text{total population}}{\text{total number of seats}}$	Round each standard quota down to the nearest whole number. Initially give each group its lower quota. Give surplus items, one at a time, to the groups with the largest decimal parts.
Jefferson's	The modified divisor is less than the standard divisor.	Round each group's modified quota down to the nearest whole number. Apportion to each group its modified lower quota.
Adams's	The modified divisor is greater than the standard divisor.	Round each group's modified quota up to the nearest whole number. Apportion to each group its modified upper quota.
Webster's	The modified divisor may be less than, greater than, or equal to the standard divisor.	Round each group's modified quota to the nearest whole number. Apportion to each group its modified rounded quota.

Paradox	**Description**
Alabama Paradox	An increase in the total number of seats to be apportioned results in the loss of a seat for a state.
Population Paradox	State A loses seats to State B, even though the population of State A grew at a faster rate than that of State B.
New-States Paradox	A new state is added along with its fair share of seats, but this results in an old state losing a seat to another old state.