

MTH 110
Exam 2
Spring 2018

Justify all answers with neat and organized work. Clearly indicate your answers.
100 points possible.

The following formulas may or may not be useful.

$$C = \frac{n(n-1)}{2}$$

$$\text{Standard divisor} = \frac{\text{total population}}{\text{total number of seats}}$$

$$\text{Standard quota for a state} = \frac{\text{population of that state}}{\text{standard divisor}}$$

$$\text{Percent increase} = \frac{\text{amount of increase}}{\text{original amount}}$$

$${}_nP_r = \frac{n!}{(n-r)!} \quad {}_nC_r = \frac{n!}{(n-r)!r!}$$

A standard deck of cards has 52 cards.

There are 4 suits: spades, hearts, diamonds, clubs.

There are 13 ranks in each suit: Ace, 2, 3, 4, 5, 6, 7, 8, 9, 10, Jack, Queen, King.

The Jack, Queen, and King are called picture cards, or face cards.

The spades and clubs are black.

The hearts and diamonds are red.

$ABCD = BACD$	“order doesn’t matter”	combinations
$ABCD \neq BACD$	“order matters”	permutations

1. (5 pts.) Suppose that the pairwise comparison method is used to determine the winner in an election. If there are 12 candidates, how many comparisons must be made?

2. (25 pts.) The preference table for an election is shown.

Number of Votes	150	60	45	30
1st choice	D	C	C	A
2nd choice	A	A	A	D
3rd choice	B	B	D	B
4th choice	C	D	B	C

(a) Suppose the Borda count method is used. Who is the winner? (Show all work, including point totals.)

(b) Is the majority criterion satisfied? Using one or more complete sentences, explain your answer.

3. (25 pts.) A small country has 57 seats in the congress, divided among the four states, according to their respective populations. The table shows each state's population.

State	A	B	C	D	Total
Population	10,080	9120	8080	1220	28,500

(a) Apportion the seats using Hamilton's method.

(b) Suppose the number of seats is increased from 57 to 58. Use Hamilton's method to reapportion the seats.

(c) What paradox occurs? Explain briefly.

4. (5 pts.) Nineteen bands have volunteered to perform at a concert, but there is only enough time for four of them to play. How many different lineups are possible?

5. (5 pts.) From the 27 books that you've bought during the past year, you plan to take 4 with you on vacation. How many different sets of four books can you take?

6. (5 pts.) An ice cream store sells three drinks (sodas, smoothies, or milk shakes), in four sizes (small, medium, large, and dino-size), and three flavors (vanilla, strawberry, or chocolate). In how many ways can a customer order a drink?

7. (5 pts.) License plates in a particular state display three letters followed by two numbers, such as CAT-88 or WBB-01. How many different license plates can be manufactured for this state?

8. (5 pts.) In how many ways can a committee of three men and six women be formed from a group of eight men and nine women?

9. (5 pts.) Compute: $\frac{24!}{21!}$

10. (5 pts.) Compute: $\frac{{}_6C_3 \cdot {}_8C_5}{{}_{14}C_8}$

11. (5 pts.) You are dealt one card from a 52-card deck. Find the probability of being dealt a black seven.

12. (5 pts.) A single die is rolled twice. The 36 equally likely outcomes are shown as follows.

	1	2	3	4	5	6
1	(1, 1)	(1, 2)	(1, 3)	(1, 4)	(1, 5)	(1, 6)
2	(2, 1)	(2, 2)	(2, 3)	(2, 4)	(2, 5)	(2, 6)
3	(3, 1)	(3, 2)	(3, 3)	(3, 4)	(3, 5)	(3, 6)
4	(4, 1)	(4, 2)	(4, 3)	(4, 4)	(4, 5)	(4, 6)
5	(5, 1)	(5, 2)	(5, 3)	(5, 4)	(5, 5)	(5, 6)
6	(6, 1)	(6, 2)	(6, 3)	(6, 4)	(6, 5)	(6, 6)

Find the probability of getting two numbers whose sum is 8.

Voting Method	How the Winning Candidate Is Determined
Plurality Method	The candidate with the most first-place votes is the winner.
Borda Count Method	Voters rank all candidates from the most favorable to the least favorable. Each last-place vote receives 1 point, each next-to-last-place vote 2 points, and so on. The candidate with the most points is the winner.
Plurality-with-Elimination Method	The candidate with the majority (over 50%) of first-place votes is the winner. If no candidate receives a majority, eliminate the candidate with the fewest first-place votes. Either hold another election or adjust the preference table. Continue this process until a candidate receives a majority of first-place votes. That candidate is the winner.
Pairwise Comparison Method	Voters rank all the candidates. A series of comparisons is made in which each candidate is compared to each of the other candidates. The preferred candidate in each comparison receives 1 point; in case of a tie, each receives $\frac{1}{2}$ point. The candidate with the most points is the winner.

Fairness Criterion	Description
Majority Criterion	If a candidate receives a majority of first-place votes in an election, then that candidate should win the election.
Head-to-Head Criterion	If a candidate is favored when compared head-to-head with every other candidate, then that candidate should win the election.
Monotonicity Criterion	If a candidate wins an election and, in a reelection, the only changes are changes that favor the candidate, then that candidate should win the reelection.
Irrelevant Alternatives Criterion	If a candidate wins an election and, in a recount, the only changes are that one or more of the other candidates are removed from the ballot, then that candidate should still win the election.

Method	Divisor	Apportionment
Hamilton's	Standard divisor $= \frac{\text{total population}}{\text{total number of seats}}$	Round each standard quota down to the nearest whole number. Initially give each group its lower quota. Give surplus items, one at a time, to the groups with the largest decimal parts.
Jefferson's	The modified divisor is less than the standard divisor.	Round each group's modified quota down to the nearest whole number. Apportion to each group its modified lower quota.
Adams's	The modified divisor is greater than the standard divisor.	Round each group's modified quota up to the nearest whole number. Apportion to each group its modified upper quota.
Webster's	The modified divisor may be less than, greater than, or equal to the standard divisor.	Round each group's modified quota to the nearest whole number. Apportion to each group its modified rounded quota.

Paradox	Description
Alabama Paradox	An increase in the total number of seats to be apportioned results in the loss of a seat for a state.
Population Paradox	State A loses seats to State B, even though the population of State A grew at a faster rate than that of State B.
New-States Paradox	A new state is added along with its fair share of seats, but this results in an old state losing a seat to another old state.