

MTH 110
Quiz 2, Form A
Spring 2016

Justify all answers with neat and organized work. Clearly indicate your answers.
20 points possible.

The following formulas may or may not be useful.

$$C = \frac{n(n-1)}{2}$$

$$\text{Standard divisor} = \frac{\text{total population}}{\text{total number of seats}}$$

$$\text{Standard quota for a state} = \frac{\text{population of that state}}{\text{standard divisor}}$$

$$\text{Percent increase} = \frac{\text{amount of increase}}{\text{original amount}}$$

1. (4 pts.) The preference table for an election is shown.

Number of Votes	30	24	15	6
1st choice	X	W	W	Y
2nd choice	Y	Y	Z	Z
3rd choice	Z	Z	X	X
4th choice	W	X	Y	W

Suppose the Borda count method is used. Who is the winner? (Show all work.)

2. (6 pts.) The preference table for an election is shown.

Number of Votes	90	66	60	36	6
1st choice	H	E	F	G	G
2nd choice	G	F	E	E	F
3rd choice	F	G	G	F	H
4th choice	E	H	H	H	E

(a) Suppose the plurality-with-elimination method is used. Who is the winner? (Show all work.)

(b) Suppose that candidate H drops out, but the winner is still chosen using the plurality-with-elimination method. Who is the winner? (Show all work.)

(c) Is the irrelevant alternatives criterion satisfied? Using one or more complete sentences, explain your answer.

3. (4 pts.) A country is composed of six states, A, B, C, D, E, and F. The population of each state, in thousands, is given in the following table.

State	A	B	C	D	E	F
Population (in thousands)	454	892	676	912	1052	1670

Use Adams's method with a modified divisor of $d = 32$ to apportion 180 seats among the states. (Show the modified quotas as part of your work.)

4. (6 pts.) A small country has 57 seats in the congress, divided among four states according to their respective populations. The population of each state, in thousands, is given in the following table.

State	A	B	C	D
Population (in thousands)	305	2280	2520	2020

(a) Apportion the 57 seats using Hamilton's method. (Show all work.)

(b) Suppose the number of seats is increased from 57 to 58. Use Hamilton's method to apportion the 58 seats. (Show all work.)

(c) Which paradox occurs? Explain briefly.

Voting Method	How the Winning Candidate Is Determined
Plurality Method	The candidate with the most first-place votes is the winner.
Bourda Count Method	Voters rank all candidates from the most favorable to the least favorable. Each last-place vote receives 1 point, each next-to-last-place vote 2 points, and so on. The candidate with the most points is the winner.
Plurality-with-Elimination Method	The candidate with the majority (over 50%) of first-place votes is the winner. If no candidate receives a majority, eliminate the candidate with the fewest first-place votes. Either hold another election or adjust the preference table. Continue this process until a candidate receives a majority of first-place votes. That candidate is the winner.
Pairwise Comparison Method	Voters rank all the candidates. A series of comparisons is made in which each candidate is compared to each of the other candidates. The preferred candidate in each comparison receives 1 point; in case of a tie, each receives $\frac{1}{2}$ point. The candidate with the most points is the winner.

Fairness Criterion	Description
Majority Criterion	If a candidate receives a majority of first-place votes in an election, then that candidate should win the election.
Head-to-Head Criterion	If a candidate is favored when compared head-to-head with every other candidate, then that candidate should win the election.
Monotonicity Criterion	If a candidate wins an election and, in a reelection, the only changes are changes that favor the candidate, then that candidate should win the reelection.
Irrelevant Alternatives Criterion	If a candidate wins an election and, in a recount, the only changes are that one or more of the other candidates are removed from the ballot, then that candidate should still win the election.

Method	Divisor	Apportionment
Hamilton's	Standard divisor $= \frac{\text{total population}}{\text{total number of seats}}$	Round each standard quota down to the nearest whole number. Initially give each group its lower quota. Give surplus items, one at a time, to the groups with the largest decimal parts.
Jefferson's	The modified divisor is less than the standard divisor.	Round each group's modified quota down to the nearest whole number. Apportion to each group its modified lower quota.
Adams's	The modified divisor is greater than the standard divisor.	Round each group's modified quota up to the nearest whole number. Apportion to each group its modified upper quota.
Webster's	The modified divisor may be less than, greater than, or equal to the standard divisor.	Round each group's modified quota to the nearest whole number. Apportion to each group its modified rounded quota.

Paradox	Description
Alabama Paradox	An increase in the total number of seats to be apportioned results in the loss of a seat for a state.
Population Paradox	State A loses seats to State B, even though the population of State A grew at a faster rate than that of State B.
New-States Paradox	A new state is added along with its fair share of seats, but this results in an old state losing a seat to another old state.