MTH 110
Exam 2, Form B
Spring 2016

Justify all answers with neat and organized work. Clearly indicate your answers. 100 points possible.

The following formulas may or may not be useful.

$$C = \frac{n(n-1)}{2}$$

$$\text{Standard divisor} = \frac{\text{total population}}{\text{total number of seats}}$$

$$\text{Standard quota for a state} = \frac{\text{population of that state}}{\text{standard divisor}}$$

$$\text{Percent increase} = \frac{\text{amount of increase}}{\text{original amount}}$$

$$_nP_r = \frac{n!}{(n-r)!} \qquad _nC_r = \frac{n!}{(n-r)!r!}$$

A standard deck of cards has 52 cards.
There are 4 suits: spades, hearts, diamonds, clubs.
There are 13 ranks in each suit: Ace, 2, 3, 4, 5, 6, 7, 8, 9, 10, Jack, Queen, King.

The Jack, Queen, and King are called picture cards, or face cards.
The spades and clubs are black.
The hearts and diamonds are red.

---

1. (5 pts.) Suppose that the pairwise comparison method is used to determine the winner in an election. If there are 13 candidates, how many comparisons must be made?

2. (25 pts.) The preference table gives the results of a straw vote among three candidates A, B, and C.

Number of Votes	40	32	28	16
1st choice	F	H	G	G
2nd choice	G	F	H	F
3rd choice	H	G	F	H

(a) Suppose the plurality-with-elimination method is used. Who is the winner of the straw vote?

(b) In the actual election, the four voters in the last column who voted GFH, in that order, change their votes to FGH. Using the plurality-with-elimination method, who wins the actual election? Is the monotonicity criterion satisfied? Using one or more complete sentences, explain your answer.

3. (25 pts.) A small country has 30 seats in the congress, divided among three states according to their respective populations. The population of each state, in thousands, is given in the following table.

Course	College Algebra	Statistics	Liberal Arts Math	Total
Enrollment	978	500	322	1800

(a) Apportion the 30 seats using Hamilton's method.

(b) Suppose the number of seats is increased from 30 to 31. Use Hamilton's method to reapportion the seats.

(c) What paradox occurs? Explain briefly.

4. (5 pts.) Suppose you are asked to list, in order of preference, the six favorite songs you purchased in the past 12 months. If you bought 15 songs over this time period, in how many ways can the five favorite be ranked?

5. (5 pts.) An election ballot asks voters to select five city commissioners from a group of thirteen candidates. In how many ways can this be done?

6. (5 pts.) In how many ways can eight airplanes line up for departure on a runway?

7. (5 pts.) You can purchase a particular model of a new car with a choice of nine colors, with or without automatic transmission, with or without four-wheel drive, and with two, three, or four speakers. In how many different ways can you order the car?

8. (5 pts.) In how many distinct ways can the letters of the word LOLLAPALOOZA be arranged?

9. (5 pts.) Compute: $\dfrac{17!}{11!}$

10. (5 pts.) Compute: $\dfrac{{}_8C_3}{{}_6C_4}$

11. (5 pts.) You are dealt one card from a 52-card deck. Find the probability of being dealt a card greater than 3 and less than 9.

12. (5 pts.) A single die is rolled twice. The 36 equally likely outcomes are shown as follows.

	1	2	3	4	5	6
1	$(1,1)$	$(1,2)$	$(1,3)$	$(1,4)$	$(1,5)$	$(1,6)$
2	$(2,1)$	$(2,2)$	$(2,3)$	$(2,4)$	$(2,5)$	$(2,6)$
3	$(3,1)$	$(3,2)$	$(3,3)$	$(3,4)$	$(3,5)$	$(3,6)$
4	$(4,1)$	$(4,2)$	$(4,3)$	$(4,4)$	$(4,5)$	$(4,6)$
5	$(5,1)$	$(5,2)$	$(5,3)$	$(5,4)$	$(5,5)$	$(5,6)$
6	$(6,1)$	$(6,2)$	$(6,3)$	$(6,4)$	$(6,5)$	$(6,6)$

Find the probability of getting two numbers whose sum is 6.

Voting Method	How the Winning Candidate Is Determined
Plurality Method	The candidate with the most first-place votes is the winner.
Bourda Count Method	Voters rank all candidates from the most favorable to the least favorable. Each last-place vote receives 1 point, each next-to-last-place vote 2 points, and so on. The candidate with the most points is the winner.
Plurality-with-Elimination Method	The candidate with the majority (over 50%) of first-place votes is the winner. If no candidate receives a majority, eliminate the candidate with the fewest first-place votes. Either hold another election or adjust the preference table. Continue this process until a candidate receives a majority of first-place votes. That candidate is the winner.
Pairwise Comparison Method	Voters rank all the candidates. A series of comparisons is made in which each candidate is compared to each of the other candidates. The preferred candidate in each comparison receives 1 point; in case of a tie, each receives $\frac{1}{2}$ point. The candidate with the most points is the winner.

Fairness Criterion	Description
Majority Criterion	If a candidate receives a majority of first-place votes in an election, then that candidate should win the election.
Head-to-Head Criterion	If a candidate is favored when compared head-to-head with every other candidate, then that candidate should win the election.
Monotonicity Criterion	If a candidate wins an election and, in a reelection, the only changes are changes that favor the candidate, then that candidate should win the reelection.
Irrelevant Alternatives Criterion	If a candidate wins an election and, in a recount, the only changes are that one or more of the other candidates are removed from the ballot, then that candidate should still win the election.

Method	**Divisor**	**Apportionment**
Hamilton's	Standard divisor $= \dfrac{\text{total population}}{\text{total number of seats}}$	Round each standard quota down to the nearest whole number. Initially give each group its lower quota. Give surplus items, one at a time, to the groups with the largest decimal parts.
Jefferson's	The modified divisor is less than the standard divisor.	Round each group's modified quota down to the nearest whole number. Apportion to each group its modified lower quota.
Adams's	The modified divisor is greater than the standard divisor.	Round each group's modified quota up to the nearest whole number. Apportion to each group its modified upper quota.
Webster's	The modified divisor may be less than, greater than, or equal to the standard divisor.	Round each group's modified quota to the nearest whole number. Apportion to each group its modified rounded quota.

Paradox	**Description**
Alabama Paradox	An increase in the total number of seats to be apportioned results in the loss of a seat for a state.
Population Paradox	State A loses seats to State B, even though the population of State A grew at a faster rate than that of State B.
New-States Paradox	A new state is added along with its fair share of seats, but this results in an old state losing a seat to another old state.