MTH 110
Exam 2
Fall 2024

Show work. Be neat and organized. Clearly indicate your answers.
100 points possible.

1. (15 pts.) A class is given the option of choosing a day for the final exam. The students in the class are asked to rank the three available days, Monday (M), Wednesday (W), and Friday (F). The results of the election are shown in the following preference table.

Number of Votes	16	10	5	3
1st choice	F	F	W	M
2nd choice	W	M	F	W
3rd choice	M	W	M	F

(a) How many students voted in the election?

(b) How many students selected the days in this order: F, M, W?

(c) How many students selected Friday as their first choice for the exam date?

2. (5 pts.) Suppose that the pairwise comparison method is used to determine the winner in an election. If there are eight candidates, how many comparisons must be made? (Show work to support your answer.)

3. (10 pts.) Twenty-nine people are asked to taste-test and rank three different brands of yogurt, A, B, and C. The preference table shows the rankings of the 29 voters.

Number of Votes	15	10	4
1st choice	A	C	B
2nd choice	B	B	C
3rd choice	C	A	A

Suppose that the Borda count method is used to determine the winner. Which brand wins the taste test? (Show your work, including the Borda count points for each candidate.)

4. (20 pts.) The 87 members of an organization are meeting to elect a keynote speaker for a special event. The choices are A, B, or C. A straw vote is taken, and the results are given in the preference table below.

Number of Votes	30	24	21	12
1st choice	A	C	B	B
2nd choice	B	A	C	A
3rd choice	C	B	A	C

(a) Using the plurality-with-elimination method, which speaker wins the straw vote? (Show your work, including the results of each round.)

(b) After a lengthy discussion, the actual election is held. Everyone votes in exactly the same way except for 12 students. The 12 voters shown in the last column of the preference table, who voted B, A, C in that order, all change their ballots to A, B, C, to make A their first choice.

Construct a new preference table with this change. Using the plurality-with-elimination method, which speaker wins the actual election? (Show your work, including the results of each round.)

(c) Is the monotonicity criterion satisfied? Why or why not?

☐ Yes. The winner of the straw vote lost the actual election even though the changes were in their favor.

☐ No. The winner of the straw vote won the actual election because the changes were in their favor.

☐ Yes. The winner of the straw vote won the actual election.

☐ No. The winner of the straw vote lost the actual election even though the changes were in their favor.

5. (20 pts.) A small country is composed of four states, A, B, C, and D. The population of each state, in thousands, is given in the following table.

State	A	B	C	D	Total
Population (in thousands)	470	1068	738	1324	3600

According to the country's constitution, the congress will have 400 seats, divided among the four states according to their respective populations.

(a) Find the standard divisor, in thousands. How many people are there for each seat in congress?

(b) Find each state's standard quota. (Round to two decimal places.)

(c) Find each state's lower quota.

(d) Find each state's upper quota.

6. (10 pts.) A small country is composed of four states, A, B, C, and D. The population of each state, in thousands, is given in the following table.

State	A	B	C	D	Total
Population (in thousands)	142	234	422	2388	3186

According to the country's constitution, the congress will have 35 seats, divided among the four states according to their respective populations.

Use Adams's method with $d = 96$ to find each state's apportionment of congressional seats.

7. (20 pts.) A corporation has two branches, A and B. Each year the company awards 100 promotions within its branches. The table shows the number of employees in each branch.

Branch	A	B	Total
Employees	4725	45,275	50,000

(a) Use Hamilton's method to apportion the promotions. (Show your work, including the standard quotas.)

Branch	A	B	Total
Employees	4725	45,275	50,000

(b) Suppose that a third branch, C, with the number of employees shown in the table, is added to the corporation. The company adds 10 new yearly promotions for branch C. Use Hamilton's method to reapportion the 110 promotions. (Show your work, including the standard quotas.)

Branch	A	B	C	Total
Employees	4725	45,275	5200	55,200

(c) Does the new-states paradox occur?

☐ Yes; When branch C is added along with its fair share of promotions, branch A loses a promotion to branch B.

☐ Yes; When branch C is added along with its fair share of promotions, branch B loses a promotion to branch A.

☐ No; the new-states paradox does not occur.

Formula:

$$C = \frac{n(n-1)}{2}$$

Voting Method	**How the Winning Candidate Is Determined**
Plurality Method	The candidate with the most first-place votes is the winner.
Bourda Count Method	Voters rank all candidates from the most favorable to the least favorable. Each last-place vote receives 1 point, each next-to-last-place vote 2 points, and so on. The candidate with the most points is the winner.
Plurality-with-Elimination Method	The candidate with the majority (over 50%) of first-place votes is the winner. If no candidate receives a majority, eliminate the candidate with the fewest first-place votes. Either hold another election or adjust the preference table. Continue this process until a candidate receives a majority of first-place votes. That candidate is the winner.
Pairwise Comparison Method	Voters rank all the candidates. A series of comparisons is made in which each candidate is compared to each of the other candidates. The preferred candidate in each comparison receives 1 point; in case of a tie, each receives $\frac{1}{2}$ point. The candidate with the most points is the winner.

Fairness Criterion	**Description**
Majority Criterion	If a candidate receives a majority of first-place votes in an election, then that candidate should win the election.
Head-to-Head Criterion	If a candidate is favored when compared head-to-head with every other candidate, then that candidate should win the election.
Monotonicity Criterion	If a candidate wins an election and, in a reelection, the only changes are changes that favor the candidate, then that candidate should win the reelection.
Irrelevant Alternatives Criterion	If a candidate wins an election and, in a recount, the only changes are that one or more of the other candidates are removed from the ballot, then that candidate should still win the election.

Formulas:

$$C = \frac{n(n-1)}{2}$$

$$\text{Standard divisor} = \frac{\text{total population}}{\text{total number of seats}}$$

$$\text{Standard quota for a state} = \frac{\text{population of that state}}{\text{standard divisor}}$$

$$\text{Percent increase} = \frac{\text{amount of increase}}{\text{original amount}} \times 100\%$$

Method	**Divisor**	**Apportionment**
Hamilton's	Standard divisor $= \frac{\text{total population}}{\text{total number of seats}}$	Round each standard quota down to the nearest whole number. Initially give each group its lower quota. Give surplus items, one at a time, to the groups with the largest decimal parts.
Jefferson's	The modified divisor is less than the standard divisor.	Round each group's modified quota down to the nearest whole number. Apportion to each group its modified lower quota.
Adams's	The modified divisor is greater than the standard divisor.	Round each group's modified quota up to the nearest whole number. Apportion to each group its modified upper quota.
Webster's	The modified divisor may be less than, greater than, or equal to the standard divisor.	Round each group's modified quota to the nearest whole number. Apportion to each group its modified rounded quota.

Paradox	**Description**
Alabama Paradox	An increase in the total number of seats to be apportioned results in the loss of a seat for a state.
Population Paradox	State A loses seats to State B, even though the population of State A grew at a faster rate than that of State B.
New-States Paradox	A new state is added along with its fair share of seats, but this results in an old state losing a seat to another old state.