**Math 110**
**Exam 2**
**Fall 2006**

Show all work in a neat and organized fashion. Clearly indicate your answers.

Always show enough work to demonstrate that you understand the method and are using it correctly.

100 points possible.

The following formulas may or may not be useful.

$$C = \frac{n(n-1)}{2}$$

$$\text{Percent increase} = \frac{\text{amount of increase}}{\text{original amount}}$$

$$\text{Percent decrease} = \frac{\text{amount of decrease}}{\text{original amount}}$$

$$\text{Standard divisor} = \frac{\text{total population}}{\text{total number of seats}}$$

$$\text{Standard quota for a state} = \frac{\text{population of that state}}{\text{standard divisor}}$$

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**1.** (20 pts.) (a) State the Majority Criterion.

(b) State the Monotonicity Criterion.

(c) State the Alabama Paradox.

(d) State the Population Paradox.

**2.** (10 pts.) The preference table for an election is shown.

| Number of Votes | 30 | 24 | 15 | 6 |
|---|---|---|---|---|
| 1st choice | C | B | B | D |
| 2nd choice | D | D | A | A |
| 3rd choice | A | A | C | C |
| 4th choice | B | C | D | B |

Suppose the Borda count method is used.

(a) Who is the winner? (Show enough work.)

(b) Which fairness criterion is violated? Using one or more complete sentences, explain your answer.

**3.** (20 pts.) The preference table for an election is shown.

| Number of Votes | 15 | 15 | 15 | 15 | 5 | 5 | 5 | 5 |
|---|---|---|---|---|---|---|---|---|
| 1st choice | B | C | B | C | D | D | A | C |
| 2nd choice | E | B | D | B | A | C | E | B |
| 3rd choice | A | E | E | D | C | B | B | A |
| 4th choice | D | A | A | E | E | E | D | D |
| 5th choice | C | D | C | A | B | A | C | E |

Suppose the pairwise comparison method is used.

(a) List all the comparisons that must be made.

(b) Using the pairwise comparison method, who is the winner? (Show enough work.)

(c) Suppose that candidate D drops out, but the winner is still chosen using the pairwise comparison method. List all the comparisons that must be made. Who is the winner? (Show enough work.)

(d) In part (c), which fairness criterion is violated? Using one or more complete sentences, explain your answer.

**4.** (25 pts.) A country is composed of four states, A, B, C, and D. The population of each state is given in the following table.

| State | A | B | C | D |
|---|---|---|---|---|
| Population | 1448 | 1222 | 784 | 550 |

According to the country's constitution, the congress will have 35 seats, divided among the four states according to their respective populations.

(a) Find the standard divisor.

(b) Find each state's standard quota.

(c) Find each state's lower quota.

(d) Find each state's upper quota.

(e) Use Jefferson's method with a modified divisor of $d = 111$ to find each state's apportionment of congressional seats.

**5.** (25 pts.) A small country has 24 seats in the congress, divided among three states according to their respective populations. The table shows each state's population before and after the country's population increase.

| State | A | B | C | Total |
|---|---|---|---|---|
| Original Population | 4480 | 1060 | 1980 | 7520 |
| New Population | 5140 | 1360 | 2500 | 9000 |

(a) Use Hamilton's method to apportion the 24 congressional seats using the original population.

(b) Find the percent increase, to the nearest tenth of a percent, in the population of each state.

(c) Use Hamilton's method to apportion the 24 congressional seats using the new population.

(d) What paradox occurs? Explain briefly.