Math 110
Exam 3
Fall 2005

Justify all answers with neat and organized work. Clearly indicate your answers. 100 points possible.

In problems 12 through 21, you do not have to simplify your answers. You may leave your answers in symbolic form. (If you wish to simplify any of them, that's OK too.)

The following formulas may or may not be useful.

$$\text{Percent increase} = \frac{\text{amount of increase}}{\text{original amount}}$$

$$\text{Percent decrease} = \frac{\text{amount of decrease}}{\text{original amount}}$$

$$\text{Standard divisor} = \frac{\text{total population}}{\text{total number of seats}}$$

$$\text{Standard quota for a state} = \frac{\text{population of that state}}{\text{standard divisor}}$$

$${}_nP_r = \frac{n!}{(n-r)!} \qquad {}_nC_r = \frac{n!}{(n-r)!r!}$$

A standard deck of cards has 52 cards.
There are 4 suits: spades, hearts, diamonds, clubs.
There are 13 ranks in each suit: Ace, 2, 3, 4, 5, 6, 7, 8, 9, 10, Jack, Queen, King.

The Jack, Queen, and King are called picture cards, or face cards.
The spades and clubs are black.
The hearts and diamonds are red.

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1. (12 pts.) (a) What is the Alabama paradox?

(b) What is the population paradox?

(c) What is the new-states paradox?

2. (12 pts.) An HMO has 10 doctors to be apportioned among three clinics. The HMO decides to apportion the doctors based on the average weekly patient load for each clinic, given in the table.

(a) Apportion the doctors using Hamilton's method.

Clinic	A	B	C	Total
Average Weekly Patient Load	119	165	216	500

(b) Suppose the HMO hires one new doctor, so that the number of doctors is increased from 10 to 11. Use Hamilton's method to reapportion the doctors.

Clinic	A	B	C	Total
Average Weekly Patient Load	119	165	216	500

(c) What paradox occurs? Explain briefly.

For the rest of this page, compute each expression (4 points each).

3. $\dfrac{10!}{5!}$

4. $\dfrac{405!}{403!}$

5. $(8-5)!$

6. ${}_9P_4$

7. ${}_9P_0$

8. ${}_9P_9$

9. ${}_9C_4$

10. $\dfrac{{}_{10}C_3}{{}_6C_2}$

11. $\dfrac{{}_{10}C_2 \cdot {}_6C_3}{{}_{16}C_5}$

Important: In all the remaining problems, you do not have to simplify your answers. You may leave your answers in symbolic form. (If you wish to simplify any of them, that's OK too.)

12. (4 pts.) You can purchase a particular model of a new car with a choice of six colors, with or without automatic transmission, with or without power windows, with or without air conditioning, and with or without a CD player. In how many different ways can you order the car, with regard to these options?

13. (4 pts.) Eight patients are to be scheduled for examination by one doctor on Saturday morning. How many different ways are there to schedule their examinations?

14. (4 pts.) Suppose you are asked to list, in order of preference, the ten favorite DVDs you rented in the past 6 months. If you rented 25 DVDs over this time period, in how many ways can the ten favorite be ranked?

15. (4 pts.) In how many distinct ways can the letters of the word TATTOO be arranged?

16. (4 pts.) How many different 5-card hands can be dealt from a deck that has only the black cards?

17. (4 pts.) You are dealt one card from a 52-card deck. Find the probability of being dealt a red picture card.

18. (4 pts.) When the prototype master tape was assembled by Malcolm Davies for the album *Sgt. Pepper's Lonely Hearts Club Band* on April 6, 1967, the line-up for the seven songs on side one of the LP was different from that finally chosen. At this stage, the side one line-up was 'Sgt Pepper's Lonely Hearts Club Band'; 'With A Little Help From My Friends'; 'Being For The Benefit Of Mr Kite!'; 'Fixing A Hole'; 'Lucy In The Sky With Diamonds'; 'Getting Better'; and 'She's Leaving Home.'

If these songs are rearranged into a random order and then played on a CD player, what is the probability that 'Sgt Pepper's Lonely Hearts Club Band' will be played first and 'With A Little Help From My Friends' second?

(Optional problem, worth ZERO bonus points: What was the actual line-up finally chosen for side one?)

19. (4 pts.) A lottery game is set up so that each player chooses five different numbers from 1 to 25. If the five numbers match the five numbers drawn in the lottery, the player wins (or shares) the top cash prize. What is the probability of winning the prize with one lottery ticket? What is the probability of winning the prize with 30 lottery tickets?

20. (4 pts.) A committee of eight people is to be formed from twelve teachers and five students. Find the probability of selecting six teachers and two students.

21. (4 pts.) If you are dealt 5 cards from a shuffled deck of 52 cards, find the probability of getting three aces and two jacks.